Resource Allocation in Uplink Multi-carrier MIMO systems for Low-complexity Transceivers

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***Abstract*— This paper proposes a resource-allocation method in a multi-carrier multiple-access channel with multiple antennas at the users and at the base station. Two additional constraints are imposed to reduce transceiver complexity. First, the number of users sharing a subcarrier is limited to the number of antennas at the base station, so that each receive dimension is not shared by more than one user; this reduces the decoding complexity of the receiver. Second, each user is forced to employ beam-forming by constraining its transmit covariance matrix to be rank-one. With these additional constraints, algorithms for user-selection and power allocation are presented. Numerical results show that the resulting sum-rate is very close to the sum-rate capacity and the user-selection does not affect the short-term fairness among users.**

I. Introduction

The information-theoretic capacity and optimal power allocation for multiple-access channels (MACs) have been well-studied in the literature [1][2][3]. Specifically, the sum-rate capacity of the Gaussian MAC channel is achieved by having all users transmit simultaneously with an appropriate power-allocation, and applying successive decoding at the receiver. In practice, it is desirable to limit the number of simultaneous transmissions in order to reduce the decoding complexity. However, one must be careful while selecting the active users in a MAC as the loss with respect to capacity may be very large. For example, in a fading MAC using a single carrier and single antennas at the transmitters and the receiver (i.e., a single-carrier SISO MAC), selection of a subset of users from a larger group will, in general, result in a substantial loss compared to the sum-rate capacity if a short-term power constraint is enforced (i.e., power constraint applied separately to each fading state). However, the authors in [4] and [5] showed that it is optimal to select one user to be served in each dimension[1] in a SISO MAC with inter-symbol interference (ISI) having a power constraint over subcarriers or a fading single-carrier SISO MAC with a long-term power constraint.

A general way of investigating user-selection is to consider the optimality conditions for the solution to the sum-rate capacity problem. For example, by examining the conditions for optimal power allocation, the authors in [5] showed that only one user can transmit at a time in a fading SISO MAC. The optimal power allocation that achieves the sum-rate capacity of a MIMO MAC was shown to satisfy the simultaneous water-filling conditions (SWF) in [6], where an iterative-waterfilling (IWF) algorithm was presented to obtain the optimal transmit spectra. Using the SWF conditions, the authors in [7] investigated the degrees of freedom in a MIMO MAC employing orthogonal frequency-division-multiplexing (MIMO-OFDM MAC). Dimension counting arguments in [7][8] showed that as the number of subcarriers becomes very large, the number of users that can share a dimension (frequency or time) is upper bounded by the square of the number of antennas at the base-station. In addition, it is suggested in [7] that most users employ beam-forming to achieve the sum-rate capacity when the number of users becomes large.

This paper investigates the restriction of the number of independent streams transmitted by the users in a MIMO-OFDM MAC to the number of antennas at the base station[2]. The motivation is that restricting the number of active streams at each subcarrier to the number of receive antennas would avoid sharing of a receive dimension by more than one user - this would greatly reduce the decoding complexity of the receiver. In addition, beam-forming is a simple transmission strategy that would need little feedback from the base-station compared to feeding back the entire channel matrix. Therefore, restriction of the selected users to use beam-forming is also investigated. Simulation results show that the presented algorithm results in less than a 2% loss in data rate compared to the sum-rate capacity, while achieving short-term fairness.

The remainder of the paper is organized as follows: The system model is described in Section II. Section III briefly reviews the simultaneous water-filling conditions for achieving the sum-rate capacity and the degrees of freedom of a MIMO-OFDM MAC. The algorithms for user-selection and power-allocation are described in Section IV and the fairness indices are discussed in Section V. Simulation results are presented in Section VI, and the paper is concluded in Section VII.

Notation: Upper-case letters denote matrices, and boldface lower-case letters denote vectors. $Tr(X)$ and $|X|$, respectively, denote the trace and determinant of the matrix X. diag(X) is a vector containing the diagonal elements of the matrix X. $(\cdot)^*$ denotes the Hermitian conjugate. $\mathbb{E}(\cdot)$ denotes the expectation operation. $CN(0, 1)$ denotes a complex Gaussian distribution with zero mean and unit variance. I_N denotes an $N \times N$ identity matrix.

[1] *frequency* for the case of the ISI MAC and *time* for the fading single-carrier SISO MAC

[2] The results also apply to a fading MIMO MAC with long-term power constraints.

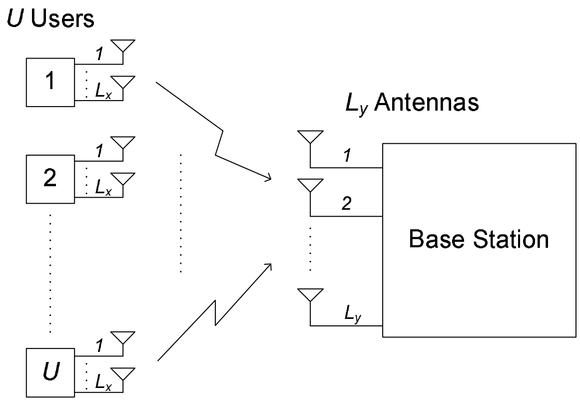

Fig. 1. A MIMO multiple-access channel

II. System Model

A Gaussian multiple-access channel is shown in Fig. 1, where the U users have L_x antennas each and the base station has L_y receive antennas. The number of subcarriers in the OFDM system is N. A block-fading model is assumed for the channel matrix of each user, i.e., the channel changes in an independent and identically distributed (i.i.d.) manner at the end of a fixed duration of time. The entries of the channel matrix for each user, across antennas and subcarriers, are assumed to have an i.i.d. $CN(0,1)$ distribution. The channel matrices of different users are also assumed to be independent. Further, the users have a short-term power constraint, i.e., the power constraint is applied separately to each fading state and allocation of power over time is not considered. The received signal on subcarrier n during the i^{th} fading state is given by

$$\mathbf{y}^n(i) = \sum_{u=1}^{U} H_u^n(i)\mathbf{x}_u^n(i) + \mathbf{z}^n(i), \quad (1)$$

where $H_u^n(i)$ is the $L_y \times L_x$ channel matrix from user u to the base station, $\mathbf{x}_u^n(i)$ is the $L_x \times 1$ vector transmitted by user u, and $\mathbf{z}^n(i)$ is the $L_y \times 1$ complex Gaussian noise vector at the receiver. The noise is assumed to be independent across subcarriers. Since a short-term power constraint P_u is applied to each user, the index i of the fading state will be suppressed for ease of representation.

III. Sum-rate capacity, Iterative Water-filling, and Degrees of Freedom

The sum-rate maximization problem for the MIMO-OFDM MAC can be formulated as the following convex optimization problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{n=1}^{N} \log_2 \left(\frac{\left| \sum_{u=1}^{U} H_u^n S_u^n H_u^{n^*} + S_z^n \right|}{|S_z^n|} \right) \\ \text{subject to} \quad & \sum_{n=1}^{N} Tr(S_u^n) \le P_u, \ \forall\ u \\ & S_u^n \succeq 0, \ \forall\ u, n \end{aligned}, \quad (2)$$

where $S_u^n = \mathbb{E}(\mathbf{x}_u^n \mathbf{x}_u^{n^*})$ and $S_z^n = \mathbb{E}(\mathbf{z}^n \mathbf{z}^{n^*})$. The KKT conditions for (2), also known as the simultaneous water-filling (SWF) conditions, are shown in [7][8] to be as follows:

$$\begin{aligned} H_u^{n^*} \left(\sum_{u=1}^{U} H_u^n S_u^n H_u^{n^*} + S_z^n \right)^{-1} H_u^n + \Phi_u^n = \lambda_u I_{L_x}, \forall u, n \\ \sum_{n=1}^{N} Tr(S_u^n) = P_u, \forall u \\ Tr(S_u^n \Phi_u^n) = 0, \forall u, n \\ S_u^n, \Phi_u^n \succeq 0, \forall\ u, n, \ \text{and } \lambda_u \ge 0, \ \forall u, \end{aligned} \quad (3)$$

where λ_u and Φ_u^n, respectively, are the dual variables corresponding to the power constraints and the positive-semidefinite constraints. These conditions can be solved iteratively using the iterative water-filling (IWF) algorithm [6] to obtain the optimal power-allocation of users over space and frequency[3]. The IWF algorithm is summarized as Algorithm 1.

Algorithm 1 The IWF Algorithm [6]

Initialize $S_u^n = 0, \ u = 1, \dots, U, \ n = 1, \dots, N$
repeat
  for u=1 to U **do**
    Set $S'^n_u = \sum_{k=1, k \neq u}^{U} H_k^n S_k^n H_k^{n^*} + S_z^n, \ \forall\ n$
    Form the whitened channel $\tilde{H}_u^n = \left(S'^n_u\right)^{-1} H_u^n, \ \forall n$
    Obtain the singular value decomposition of $\tilde{H}_u^n = \mathbf{U}^n \mathbf{\Lambda}^n \mathbf{V}^{n^*}, \ \forall n$
    Concatenate the singular values across subcarriers, [diag($\mathbf{\Lambda}^n$), $\forall n$], and perform water-filling
    Update the transmit covariance matrices of user u, $S_u^n, \ \forall n$
  end for
until The sum-rate converges

Let r_u^n denote the rank of user u's optimal transmit covariance matrix, S_u^n, on subcarrier n. By counting the number of equations and variables in the SWF conditions, the authors in [7][8] showed that $\sum_{u=1}^{U} (r_u^n)^2 \le L_y^2, \ \forall\ n$ as $N \to \infty$. Consequently, the maximum number of users simultaneously transmitting on subcarrier n is L_y^2 since the minimum rank of a user's transmitted signal is 1.

Further, for the case of a single-carrier MIMO MAC, [7] showed using a similar dimension counting argument that most users will transmit with a rank-one covariance matrix when the number of users is much larger than the number of receive antennas. In other words, they will employ beam-forming and still achieve the sum-rate capacity. It was also observed empirically in [7] that most users employed beam-forming to achieve the sum-rate capacity in a MIMO-OFDM MAC.

IV. User-selection and power-allocation Algorithms

Motivated by the bound on the number of simultaneously active users and the optimality of beam-forming for achieving

[3] IWF can also be applied over time if long-term power constraints are considered.

the sum-rate capacity, this section considers the problem of restricting the number of users sharing a subcarrier to L_y, while each user employs transmit beam-forming. With additional constraints, the sum-rate maximization problem is formulated as follows:

$$\begin{aligned} \text{maximize} \quad & \sum_{n=1}^{N} \log_2 \left(\frac{\left| \sum_{u=1}^{L_y} H^n_{\pi_n(u)} S^n_{\pi_n(u)} H^{n^*}_{\pi_n(u)} + S^n_z \right|}{|S^n_z|} \right) \\ \text{subject to} \quad & \sum_{n=1}^{N} Tr(S^n_u) \leq P_u, \ \forall \ u \\ & S^n_u \succeq 0, \ \forall \ u, n \\ & \text{rank}(S^n_u) \leq 1, \ \forall \ u, n \end{aligned} \tag{4}$$

where $\pi_n(u)$ is the u^{th} user in a selection of L_y users from U users for subcarrier n and the rank-one constraints reflect that users can only employ transmit beam-forming. Problem (4) is a non-convex optimization problem because of the combinatorial user-selection constraints and the rank constraints on the users' transmit covariance matrices. The following subsections present a suboptimal algorithm for the problem, which nevertheless has very little rate-loss compared to the sum-rate capacity.

A. User-Selection

Let us consider the optimal power allocation for the sum-rate capacity problem (2), which is obtained by the IWF algorithm. The rate-tuple on each subcarrier n will correspond to a corner point of the polytope in the capacity region, which is obtained using different successive-decoding orders and the optimal transmit covariance matrices of the users on subcarrier n [6]. Assume that as a result of IWF, K ($> L_y$) users share subcarrier n. Without loss of generality, the K users can be labeled as users $1, \dots, K$. Proposition 1 first considers the selection of the $K-1$ out of K users that achieve the maximum sum-rate.

Proposition 1: In a single-carrier MIMO MAC with K (> 1) users having fixed transmit covariance matrices $S_k, \ k = 1, \dots, K$, the $K-1$ users that maximize the sum-rate can be obtained by removing the user u that satisfies

$$u = \arg\max_{1 \leq k \leq K} \left| \sum_{\substack{k=1 \\ k \neq u}}^{K} H_k S_k H_k^* + S_z \right|. \tag{5}$$

The sum-rate after removing user v is

$$R^{\text{sum}}_{\text{remove } v} = \log_2 \left(\frac{\left| \sum_{\substack{k=1 \\ k \neq v}}^{K} H_k S_k H_k^* + S_z \right|}{|S_z|} \right), \tag{6}$$

and hence Proposition 1 follows. In addition, the data rate of user u obtained by treating other users as noise is

$$R'_u = \log_2 \left(\frac{\left| \sum_{k=1}^{K} H_k S_k H_k^* + S_z \right|}{\left| \sum_{\substack{k=1 \\ k \neq u}}^{K} H_k S_k H_k^* + S_z \right|} \right). \tag{7}$$

Hence, Proposition 1 has the interpretation that the user who has the lowest data rate when treating the other users as noise should be removed.

Let the set of users sharing tone n be $\mathcal{J}^n \subseteq \{1, \dots, U\}$, and let the number of users sharing tone n be K_n. Using Proposition 1, the following greedy user-selection algorithm is proposed:

Algorithm 2 Geedy User-selection

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1: for n=1 to N do
2:    while K_n > L_y do
3:       Choose the user u ∈ J_n with the lowest data rate
         when treating the other K_n − 1 users as noise
4:       Remove user u from tone n, J_n ← J_n\{u}, K_n ←
         K_n − 1
5:    end while
6: end for
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While this greedy method is suboptimal in general, it only requires a total complexity of $\sum_{k=L_y+1}^{K_n} k = \frac{K_n(K_n+1)}{2} - \frac{L_y(L_y+1)}{2}$ searches on subcarrier n, thus avoiding an exhaustive search over $\binom{K_n}{L_y}$ combinations of user-selections. Simulations showed that the greedy user-selection algorithm obtained the optimal user-selection with high probability. Intuitively, this can be explained as follows: At each step of the greedy algorithm, the user that contributes least to the sum-rate is removed. If the optimal user-selection deviates from the greedy solution, then it must have retained such low-contribution users. However, this means that to obtain the optimal sum-rate, there must be a few users which are causing dominant interference to the low-contribution users. When these dominant interferers are removed, the sum-rate of other users improves significantly. However, with the assumption of i.i.d. Gaussian statistics for the channels, the probability of occurrence of such dominant interferers is small. Therefore, the greedy method mostly picks the optimal user-selection, given fixed transmit covariance matrices.

After user-selection, the users that are removed from some subcarriers will not be using their full transmit power. In addition, the transmit covariance matrices are no longer optimal after the user-selection. Hence, the overall sum-rate can be improved by re-allocating the unused power to the other subcarriers. Unfortunately, the resulting power allocation will change the shape of the capacity-region polytope at each subcarrier and, consequently, the selected users may not result in the optimal sum-rate. Nevertheless, one can think of heuristic methods to distribute the unused power of a user over the subcarriers where it is selected, while achieving good performance.

B. Power-Tightening

The unused power of the users can be re-allocated by executing the IWF algorithm again using the selected users in each subcarrier. The remaining constraint from problem (4) that needs to be satisfied is the beam-forming constraint. This constraint can be incorporated by modifying the IWF procedure (Algorithm 1) to include an additional eigen-mode selection step following Step 8. Essentially, after a user performs water-filling considering the other users' signals as noise, the largest eigen-mode of the user's transmit covariance on each subcarrier is selected and is scaled to satisfy the power that was allocated to that subcarrier. The combination of IWF, user-selection, and modified IWF is referred to as the Double-IWF algorithm and is summarized in Algorithm 3.

Algorithm 3 Double-IWF Algorithm

1: Run IWF (Algorithm 1) and obtain the optimal transmit covariance matrices to achieve the sum-rate capacity
2: Select up to L_y users on each subcarrier that maximize the sum-rate with the same transmit covariance matrices (Algorithm 2)
3: Run the modified IWF algorithm, using the selected users, that chooses the largest eigen-mode for each selected user in each subcarrier after the water-filling process.

Alternately, a simple method for re-allocating the unused power of a user after selecting the L_y users on each subcarrier would be to equally distribute the unused power for each user over the subcarriers where it was selected. This would essentially result in a simple scaling of the users' transmit covariance matrices. This method is referred to as Power-scaling. The beam-forming constraint is not necessarily satisfied by this simple method (unless eigen-mode selection is performed again similar to Double-IWF). However, the simulation results will show that Double-IWF performs better than the simple Power-scaling method although the latter ignores the beam-forming constraint.

V. Fairness considerations

Since the presented algorithm selects a subset of users on a subcarrier at each fading state, the fairness of the algorithm needs to be investigated. With a large number of receive dimensions over subcarriers and antennas (NL_y) and the i.i.d. distribution of the users' channels, one would expect the user-selection and power-allocation algorithms to provide reasonable short-term fairness. In order to quantify this, the *sliding window method* is employed [9].

Let us consider many fading realizations over time. A sliding window of size W_s is moved over the fading states, and the fading states within the window are referred to as a snapshot. This window size indicates the time interval that one needs to maintain fairness among users. For a fixed value of W_s, fairness indices are calculated for each snapshot and they are averaged over the different sliding snapshots. This process is repeated with increasing window sizes. The plot of the average fairness indices versus window size will show the fairness of the algorithm.

The Jain fairness index [9] for a given window size W_s is defined as

$$F_J(W_s) = \frac{\left(\sum_{u=1}^{U} \gamma_u(W_s)\right)^2}{U \sum_{u=1}^{U} \gamma_u^2(W_s)}, \tag{8}$$

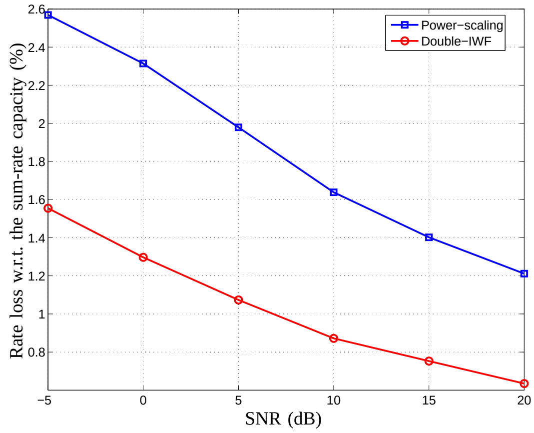

Fig. 2. Percentage sum-rate loss versus SNR. $L_x = L_y = 3$, $U = 10$, $N = 20$

where $\gamma_u(W_s)$ is the fraction of the NL_yW_s dimensions that the user u occupies at a certain snapshot. The Jain index is 1 when $\gamma_u(W_s)$ has a uniform distribution over u, which indicates absolute fairness.

The Kullback Leibler distance [1] can also be used as another fairness index where the empirical distribution $\gamma_u(W_s)$ over u can be compared with a uniform distribution over the users. The resulting index is

$$D(W_s) = \left(\sum_{u=1}^{U} \gamma_u(W_s) \log_2 \gamma_u(W_s)\right) + \log_2 U, \tag{9}$$

where an index of 0 represents absolute fairness.

VI. Simulation Results

Monte Carlo simulations results are presented in this section, where channels are randomly generated from an i.i.d. $CN(0, 1)$ distribution. The energy constraint for each user, normalized by the number of subcarriers, is assumed to be 1. The percentage loss of the algorithms with respect to the sum-rate capacity is averaged over 500 channel realizations.

Figure 2 shows that the sum-rate loss of the presented algorithms decreases as the signal-to-noise-ratio (SNR) increases because at high SNRs, the data rate of the user decoded last in successive decoding will be much larger than the other users, so adding more users does not significantly improve the sum-rate. Compared to the simple Power-scaling method, which allows the users to employ transmission schemes that are more complicated than beam-forming, the Double-IWF algorithm performs better although it restricts the users to employ only beam-forming. Moreover, the Double-IWF algorithm approaches the sum-rate capacity within 2% for a wide range of SNRs.

Figure 3 shows that the rate-loss increases as the number of antennas at the base station increases. However, the Double-IWF algorithm still has a loss less than 2% even when there are 6 receive antennas. The additional loss with larger L_y can be attributed to the transmit beam-forming constraints. As the total number of subcarriers increases, the figure also shows that

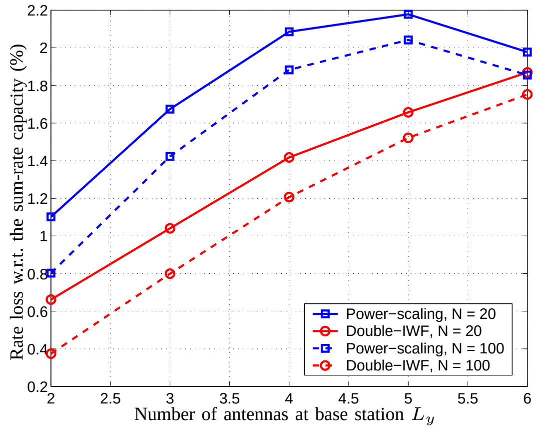

Fig. 3. Percentage sum-rate loss versus number of receive antennas. $L_x = 2$, $U = 10$, SNR = 10 dB

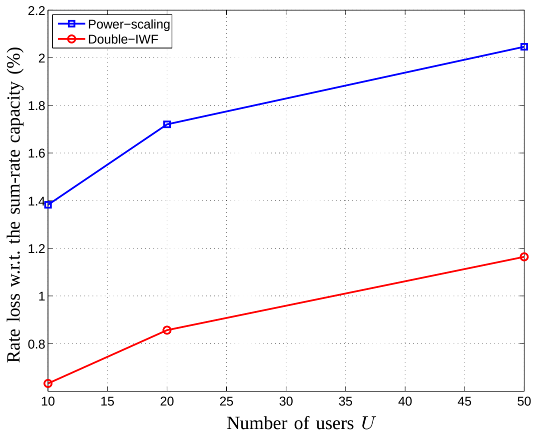

Fig. 4. Percentage sum-rate loss versus number of users. $L_x = L_y = 3$, $N = 100$, SNR = 10 dB

the rate-loss of the two algorithms is reduced. This is because there are more available dimensions for the same number of users, hence the sum-rate achieving scheme itself will try to avoid sharing many users in a subcarrier by allocating users to subcarriers where they have a larger channel SNR. As the number of users increases, the sum-rate loss increases as shown in Fig. 4 because there is more competition for the NL_y receive dimensions. In general, limiting the number of users causes little loss when the receive dimensions are plentiful.

Finally, Fig. 5 plots the Jain and Kullback Leibler fairness indices for the double-IWF algorithm. The Jain index is more than 0.96 and the KL index is less than 0.03 even for a window size of 1. Therefore, both indices strongly indicate that all users are equally sharing the available receive dimensions.

VII. Conclusions and Future Work

A near-optimal resource allocation scheme was proposed for a MIMO-OFDM MAC in order to reduce transceiver complexity. The number of users at each subcarrier is limited to the number of antennas at the receiver to prevent dimension sharing among users; each user is forced to employ transmit beam-forming to simplify the transmitters. Even with these

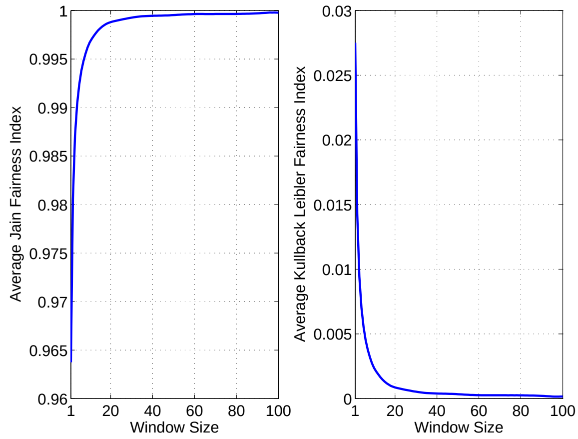

Fig. 5. The Jain and KL fairness indices show that short-term fairness is achieved. $L_x = L_y = 3$, $U = 10$, $N = 20$, SNR = 10 dB

additional constraints, the sum-rate loss compared to the sum-rate capacity is less than 2% by employing a greedy user-selection method and IWF-based power-allocation. In addition, limiting the number of users does not affect the short-term fairness among users.

The combination of transmit beam-forming and user-selection limits the number of active streams to the number of receive antennas, thereby reducing the decoding complexity of the multi-user receiver. Further reduction in complexity is possible by employing linear receivers enabling a low-complexity but high-performance solution for wireless uplinks using MIMO-OFDM.

Acknowledgements

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