Distributed Adaptive Bit-loading for Spectrum Optimization in Multi-user Multicarrier Systems

Sumanth Jagannathan, John M. Cioffi
Department of Electrical Engineering, Stanford University, Stanford, CA 94305
{sumanthj, cioffi}@stanford.edu

Abstract— **This paper presents a discrete bit-loading algorithm for multi-user multicarrier systems with application to spectrum balancing in digital-subscriber-lines. The algorithm can be implemented in a distributed manner using limited information that is sent to modems by a spectrum management center (SMC). The SMC first classifies users as strong and weak. Strong users then form a rate-penalty table that is used along with the Levin-Campello bit-loading algorithm to limit the interference to weak users. Simulation results show that the proposed algorithm can achieve near-optimal performance. Moreover, the structure of the algorithm enables easy implementation of polite bit and gain adaptation procedures without the need for executing the entire bit-loading algorithm again.**

I. Introduction

Optimization of spectra among multiple users is an important component of modern communication systems. One example is the digital-subscriber-line (DSL) network where Level 2 Dynamic Spectrum Management (DSM) methods [1][2] (also known as spectrum balancing) optimize the spectra of users to reduce interference. For example, the distributed Iterative water filling (IWF) algorithm [3] obtains significantly higher data rates than currently-used static spectrum management methods. However, IWF is sub-optimal in near/far scenarios such as upstream VDSL and central-office (CO)/remote-terminal (RT) mixed-binder downstream situations[1].

Optimal Spectrum Balancing (OSB) provides the optimal solution to the spectrum balancing problem [4]. However, its exponential complexity in the number of users and centralized nature make it prohibitive for practical implementation. Iterative Spectrum Balancing (ISB) [5][6] reduces the complexity but is still centralized. Unlike OSB and ISB, a continuous bit-loading method called SCALE was presented in [7] by applying a convex relaxation technique to spectrum balancing. With SCALE as a starting point, [8] employs a greedy method to generate a discrete bit-loading for use in practical systems.

In pursuit of a more distributed but near-optimal solution, the band preference methodology was proposed in [9] where modems independently perform waterfilling based on the power-spectral-density (PSD) masks or water-level scaling factors [10][11] provided by the spectrum management center (SMC). Autonomous Spectrum Balancing (ASB) [12] is also a distributed algorithm where information about a typical line in the network is initially passed from the SMC to the modems, which then independently perform their optimization.

[1]Refer to Fig. 2. A fiber-fed RT causes severe crosstalk to a user served by a CO, while the RT user does not experience much interference from CO.

Previous work on spectrum balancing has not considered adaptation to slow channel/noise variations by adjusting the bits and energies (or gains) of the subchannels[2]. However, such adaptation is very important since the spectra are optimized for a specific channel and noise situation at initialization, but the lines could experience variations in the noise spectrum during online operation. Such variations could be caused by other DSLs in the binder turning on/off, changing temperature and soil conditions, or simply other non-crosstalk noise sources (TV, microwave, etc.) that vary with time. In the single-user case, the optimal discrete bit-loading algorithm known as Levin-Campello (LC) [13][14] allowed for easy adaptation [15]. Although [16][17] extended the LC algorithm to the multi-user case, the extension is centralized and not amenable to bit and gain changes.

This paper uses the LC algorithm along with a rate-penalty table to reduce the amount of interference caused by strong users to weak users. The penalty table represents the reduction in the data rate of the weak user when the strong user adds more bits to a subchannel. This table is formed using information sent by the SMC to the modems. An important contribution is the development of bit and gain adaptation procedures that are polite toward weak users using the proposed framework.

II. System Model and Problem Definition

A multi-user multicarrier system with N subchannels and U users is considered with one transmitter and receiver for each user. Assuming there is no intercarrier interference and that the users are synchronized, the channel can be modeled as N parallel and independent subchannels, each of which is an interference channel with U users. Such a model can be applied independently to the upstream and downstream of a DSL system that employs frequency-division duplexing, or may be applied in a full-duplex DSL system by treating the upstream and downstream as separate interfering users.

On subchannel n, the direct channel of user u is denoted as $H^n_{u,u}$, while the crosstalk channel from user v to user u is denoted as $H^n_{u,v}$. The background noise at user u's receiver on subchannel n is assumed to be Gaussian with zero mean and energy $(\sigma^n_u)^2$. Denoting the energy transmitted by user v on subchannel n as $\mathcal{E}^n_v$, the channel signal-to-interference-plus-noise ratio (SINR) of user u on subchannel n is expressed

[2]Such adjustment is referred to as bit/gain-swapping in DSL.

as

$$g_u^n = \frac{|H_{u,u}^n|^2}{\sum_{v \neq u} \mathcal{E}_v^n |H_{u,v}^n|^2 + (\sigma_u^n)^2}. \quad (1)$$

Denoting the smallest incremental unit of information that can be transmitted as β bits (typically $\beta = 1$), the number of bits for user u on subchannel n can be expressed as

$$b_u^n = \beta \Big\lfloor \frac{1}{\beta} \log_2 \Big(1 + \frac{\mathcal{E}_u^n g_u^n}{\Gamma}\Big) \Big\rfloor, \quad (2)$$

where Γ is the gap to capacity of the code [15] plus the SINR margin used for protection against unexpected noise, and $\lfloor \cdot \rfloor$ denotes the rounding down (floor) operation. The data rate for user u is then given by

$$R_u = \sum_{n=1}^{N} b_u^n. \quad (3)$$

The general goal of spectrum balancing is to allow weak users in the network to maximize their data rates, while strong users try to achieve their target data rates, causing minimal interference to the weak users. Mathematical formulation of this problem is convenient when target data rates are fixed for $U-1$ users and the data rate of the U^{th} user is to be maximized. The algorithm presented in this paper can be applied directly to the general goal, while the latter specific problem will be considered so as to compare the performance with OSB. The constraints for each user u include a total energy limit $\mathcal{E}_u = \sum_{n=1}^{N} \mathcal{E}_u^n \leq \mathcal{E}_u^{\text{tot}}$, a PSD mask $\mathcal{E}_u^n \leq C_u^n, \ \forall n$, and a maximum bit-cap for each subchannel $b_u^n \leq \bar{b}, \ \forall n$. A bit-cap, $\bar{b}^n$, that varies among the users and subchannels can also be easily incorporated into the proposed algorithm.

III. Overview of the Proposed Algorithm

Figure 1 provides an overview of the framework using which the spectrum balancing algorithm is developed in this paper. Users in the network are classified as strong and weak by the SMC based on their channel transfer functions and noise spectra. For example, in near-far situations, the near user will typically be classified as the strong user and the far user will be classified as the weak user if they both experience similar background noise spectra. In general, the SMC can perform the classification based on the actual network topology and channel/noise conditions [18].

After the classification of users, the modems of the weak users are directed to execute their normal bit-loading algorithm, which is usually a discrete-bit approximation of water-filling. On the other hand, the strong users are directed to execute a *polite* bit-loading algorithm, which causes minimal harm to the data-rate of the weak users.

The development of the strong-user's loading algorithm is motivated by a simple 2-user example with one weak and one strong user. In this case, the goal of spectrum balancing would be for the strong user to maximize the weak user's data-rate while meeting its own target data-rate. This problem can also be thought of as the strong user trying to minimize the data-rate penalty that it causes to the weak user. Section IV essentially incorporates these data-rate penalties into the loading algorithm of the strong user to ensure politeness.

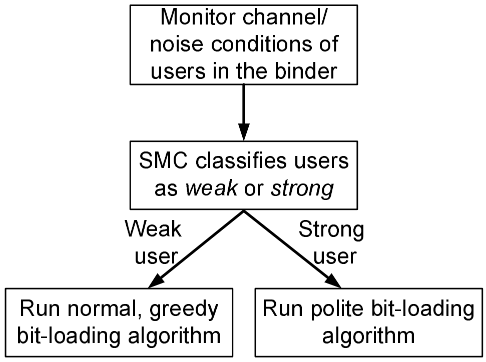

Fig. 1. Framework for near-distributed spectrum balancing

TABLE I
Incremental Energy Table for User u with $\beta = 1$

b_u^n \ n	1	2	3	$\cdots$	N
6	$e_u^1(6)$	∞	∞	$\cdots$	∞
5	$e_u^1(5)$	∞	$e_u^3(5)$	$\cdots$	∞
4	$e_u^1(4)$	$e_u^2(4)$	$e_u^3(4)$	$\cdots$	∞
3	$e_u^1(3)$	$e_u^2(3)$	$e_u^3(3)$	$\cdots$	∞
2	$e_u^1(2)$	$e_u^2(2)$	$e_u^3(2)$	$\cdots$	$e_u^N(2)$
1	$e_u^1(1)$	$e_u^2(1)$	$e_u^3(1)$	$\cdots$	$e_u^N(1)$

IV. The Distributed Discrete Bit-loading Algorithm

The discrete bit-loading algorithm presented in this paper employs a couple of tables, which are used to determine the number of bits to be loaded on each subchannel in an incremental fashion. The two tables are the energy table, which is used to determine the subchannel of least incremental energy cost where a bit can be loaded, and the rate-penalty table, which is used to determine the subchannels in which loading a bit causes minimum penalty to the data-rate of the weak users.

A. The Energy Table

Each user u in the network measures the channel SINR $g_u^n, \ \forall n$. Using (2), the minimum energy required for loading b_u^n bits, which is a multiple of β, on subchannel n is

$$\mathcal{E}_u^n(b_u^n) = \frac{\Gamma}{g_u^n}\big(2^{b_u^n} - 1\big). \quad (4)$$

The incremental energy to transmit b_u^n bits on subchannel n for user u is then defined as

$$e_u^n(b_u^n) = \begin{cases} \mathcal{E}_u^n(b_u^n) - \mathcal{E}_u^n(b_u^n - \beta) & \text{if } b_u^n > \beta \\ \mathcal{E}_u^n(b_u^n) & \text{if } b_u^n = \beta \end{cases} \quad (5)$$

$$= \frac{\Gamma}{g_u^n} 2^{b_u^n}\big(1 - 2^{-\beta}\big). \quad (6)$$

An incremental energy table such as Table I is formed by each user, which indicates the amount of energy required to

TABLE II

INCREMENTAL RATE-PENALTY TABLE FOR USER u WITH $N = 10$, $\beta = 1$

b_u^n \ n	1	2	3	4	5	6	7	8	9	10
6	1	0	1	1	1	0	0	0	0	0
5	0	1	1	0	0	0	0	0	0	0
4	0	0	0	1	0	1	0	0	0	0
3	1	1	0	0	1	0	0	0	0	0
2	0	0	1	1	0	0	0	0	0	0
1	0	0	0	0	1	1	0	0	0	0

load an extra β bits on subchannel n. When no more bits can be loaded on a subchannel because of violation of the PSD mask or bit-cap, the incremental energy is set to ∞. Such an incremental energy table is used in the optimal LC algorithms for single-user discrete bit-loading. While the gap to capacity is almost constant for many practical codes over a wide range of signal-to-noise-ratios (SNRs), there can be some deviation in practice based on the constellation size and type of code used. An advantage of the LC-based approach is that the energy table can be determined exactly by the modem based on the constellation and code that are used, instead of using the gap approximation [13].

B. The Rate-penalty Table

The SMC provides each strong user, u, with information about a weaker line, v, which it should protect. An algorithm to determine this politeness mapping between users is presented in [18]. A nominal energy-allocation, $\mathcal{E}_v^{\text{nom},n}$, is assumed for user v, which could simply be its PSD mask or could be obtained by water-filling with the nominal background noise. Using $\mathcal{E}_v^{\text{nom},n}$, the penalty table is defined as

$$B_{\text{pen},u}^n(b_u^n) = \min\left(\bar{b}, \beta\left\lfloor\frac{1}{\beta}\log_2\left(1+\frac{\mathcal{E}_v^{\text{nom},n}|H_{v,v}^n|^2}{\Gamma(\sigma_v^n)^2}\right)\right\rfloor\right) - \min\left(\bar{b}, \beta\left\lfloor\frac{1}{\beta}\log_2\left(1+\frac{\mathcal{E}_v^{\text{nom},n}|H_{v,v}^n|^2}{\Gamma\left(\mathcal{E}_u^n(b_u^n)|H_{v,u}^n|^2+(\sigma_v^n)^2\right)}\right)\right\rfloor\right),$$

and the incremental rate-penalty table is defined as

$$B_{\text{inc},u}^n(b_u^n) = \begin{cases} B_{\text{pen},u}^n(b_u^n) - B_{\text{pen},u}^n(b_u^n-\beta) & \text{if } b_u^n > \beta \\ B_{\text{pen},u}^n(b_u^n) & \text{if } b_u^n = \beta \end{cases} \tag{7}$$

for $b_u^n = \beta, 2\beta, \ldots, \bar{b}$. The remainder of the paper assumes $\beta = 1$ without loss of generality.

$B_{\text{inc},u}^n$ measures user v's incremental loss of bits caused when strong user u loads an extra bit on subchannel n. Table II shows an example of an incremental rate-penalty table. The shaded regions will be explained later. The rate-penalty table can be computed by the strong user's modem if the SMC provides it with information about the user that it needs to protect. This information would be the crosstalk channel from the strong user to the weaker user and the weaker user's background noise spectrum, where both quantities are normalized by the product of the direct channel of the weaker user and its nominal PSD divided by the gap. Alternately, the SMC could compute the rate-penalty table and send it to the strong user's modem. The information that needs to be sent to the modem can be reduced by efficiently compressing the rate-penalty table using techniques such as run-length coding [19] since the incremental rate-penalty table typically consists of a string of zeros followed by ones, especially when the crosstalk coupling is large.

C. Bit-loading Algorithm

The rate-adaptive (RA) and fixed-margin (FM) LC algorithms form an important part of the proposed spectrum balancing algorithm that is hereafter referred to as Spectrum Balancing Levin-Campello (SBLC). RA LC (Algorithm 1) maximizes the data rate subject to an energy constraint. At each iteration in RA LC, the user greedily allocates a bit to the subchannel with the least incremental energy cost, and the procedure continues until the energy, PSD, or bit-cap constraints become active. The FM version, which minimizes the energy subject to a target data-rate, is obtained by replacing the stopping criteria in step 4 of Algorithm 1 with $e_u^m(b_u^m+1) = \infty$ or $R_u \geq R_u^{\text{target}}$.

In the SBLC algorithm, the SMC first classifies the users as strong or weak. Weak users simply execute FM LC if they are provided with a target data rate, otherwise they execute RA LC. Strong users will try to attain their data-rate target while causing minimum amount of penalty to the weaker users. The users execute their algorithms independently until their PSDs converge. If the SMC classifies all users to be weak, then the users effectively participate in an IWF procedure [3].

Algorithm 1 Rate-adaptive (RA) LC algorithm for user u

1: Initialization: Form the incremental energy table e_u^n, $\forall n$. Set $\mathcal{E}_u^n \leftarrow 0$, $b_u^n \leftarrow 0$, $\forall n$. Therefore, $\mathcal{E}_u = 0$ and $R_u = 0$.
2: **loop**
3: Set $m \leftarrow \arg\min_n e_u^n(b_u^n+1)$
4: **if** $e_u^m(b_u^m+1) = \infty$ or $(\mathcal{E}_u + e_u^m(b_u^m+1)) > \mathcal{E}_u^{\text{tot}}$ **then**
5: stop **else**
6: $b_u^m \leftarrow b_u^m + 1$, $\mathcal{E}_u \leftarrow \mathcal{E}_u + e_u^m(b_u^m)$, $R_u \leftarrow R_u + 1$

Next, the polite bit-loading algorithm, which is used by the strong users, is explained. The strong user u forms the rate-penalty tables $B_{\text{pen},u}^n$ and $B_{\text{inc},u}^n$ using the parameters of the weaker user, v. Subchannels on which no rate-penalty is caused to the weaker user, $\mathcal{N}_{np} \subseteq \{1,\ldots,N\}$, are identified. For example, columns 7, 8, 9, and 10 of Table II correspond to $\mathcal{N}_{np}$. More formally, no rate-penalty means the entire column of the $B_{\text{inc},u}^n$ table is zero. FM LC is first executed on $\mathcal{N}_{np}$ to try to achieve the target data-rate without causing any data-rate penalty to the weak users. If the target data rate is not met, then the remaining subchannels $\mathcal{N}_p \subseteq \{1,\ldots,N\}\backslash\mathcal{N}_{np}$ become candidates for energy allocation. The FM LC algorithm is executed again to obtain additional data rate with a modified energy table $\tilde{e}_u^n$, where $\forall n \in \mathcal{N}_p,\ b = 1,\ldots,\bar{b}$

$$\tilde{e}_u^n(b) = \begin{cases} e_u^n(b) & \text{if } B_{\text{pen},u}^n(b) = 0 \\ \infty & \text{if } B_{\text{pen},u}^n(b) \neq 0 \end{cases}. \tag{8}$$

The first FM LC run using $\mathcal{N}_{np}$ completely avoids overlapping the strong user's PSD with that of the weaker user, while the second FM LC run using $\mathcal{N}_p$ tries to reach the target data rate by loading on the overlapping subchannels but still without penalizing the weaker user. For example, the light-shaded cells in Table II show the bit-loading achieved by the two FM LC runs. If these two passes are not sufficient to meet the target data rate, then the strong user has no option but to cause penalty to the weaker user. However, the penalty is minimized by considering the following procedure. First, the subset $\mathcal{N}_{p,\min} \subseteq \mathcal{N}_p$ of subchannels having the minimum incremental rate-penalty are considered. For these subchannels, the number of bits Δb that could be loaded before the next penalty is caused is computed. The subchannels with maximum Δb are denoted by $\mathcal{N}_{p,\min}^{\Delta b_{\max}}$, where the maximum value is $\Delta b_{\max}$. Essentially for the same amount of penalty caused, subchannels where the maximum number of bits can be loaded by the strong user are selected as candidates. For these subchannels, the remaining rate-penalty $B_{\text{rem},u}^n$ that could be caused is calculated as

$$B_{\text{rem},u}^n = \sum_{b=b_u^n+1}^{b_{\max,u}^n} B_{\text{inc},u}^n(b), \tag{9}$$

where $b_{\max,u}^n$ is the maximum number of bits that can be loaded on subchannel n by user u.

The subset of subchannels $\mathcal{N}_{p,\text{rem}} \subseteq \mathcal{N}_{p,\min}^{\Delta b_{\max}}$ having the minimum remaining penalty $B_{\text{rem},u}^n$ is then considered (shown in dark shade in Table II). Among $\mathcal{N}_{p,\text{rem}}$, the subchannel m for which the weak user's channel SNR is smallest is chosen. User u then loads an extra bit on subchannel m. The intuition is that since the weak user spends a lot of energy to load bits on subchannels with low channel SNR, the strong user can add more bits in such a subchannel if the remaining penalty is small. This could be beneficial to the weak user, which then utilizes its energy more efficiently in subchannels with better channel SNR and lower interference. The penalty procedure stops when the target data rate is met. Since the total energy constraint may have been violated during the polite bit allocation, the final routine moves bits from subchannels with the highest incremental energies to those with the lowest incremental energies until the energy constraint is satisfied. Algorithm 2 summarizes the procedure for the strong users.

The convergence properties of SBLC are beyond this short paper's scope. However, the algorithm can be verified to converge very quickly in simulations. The structure of SBLC has a complexity advantage over the ASB algorithm for discrete bit-loading. ASB for each user consists of a nesting of two loops - one to achieve the target data rate and the other to meet the energy constraint. Within the nested loop, it requires the evaluation of a Lagrangian corresponding to each integer bit-loading on each subchannel. SBLC avoids nested loops and does not require an exhaustive search over the bit-loadings.

Algorithm 2 SBLC Algorithm for Strong User u

1: Generate $B_{\text{pen},u}$, $B_{\text{inc},u}$, and identify $\mathcal{N}_{np}$, $\mathcal{N}_p$
2: Perform FM-LC on $\mathcal{N}_{np}$
3: **if** $R_u < R_u^{\text{target}}$ **then**
4:     Continue FM-LC on $\mathcal{N}_p$ with modified $\tilde{e}_u^n$ (8)
5: **while** $R_u < R_u^{\text{target}}$ **do**
6:     Identify $\mathcal{N}_{p,\text{rem}}$. Choose $m \in \mathcal{N}_{p,\text{rem}}$ with minimum channel SNR for the weak user.
7:     **if** $e_u^m(b_u^m+1) \neq \infty$ **then** $b_u^m \leftarrow b_u^m+1$, $R_u \leftarrow R_u+1$
8:     **else** remove subchannel m from $\mathcal{N}_{p,\text{rem}}$
9: **while** $\sum_{n=1}^N \mathcal{E}_u^n > \mathcal{E}_u^{\text{tot}}$ **do**
10:     $m \leftarrow \arg\max_n e_u^n(b_u^n+1)$, $p \leftarrow \arg\min_n e_u^n(b_u^n+1)$
11:     $b_u^m \leftarrow b_u^m-1$, $b_u^p \leftarrow b_u^p+1$

V. Adaptation to Channel/Noise Variations

The SBLC algorithm allows modems to adapt easily and quickly to changes in the channel SINR through the exchange of bits and energies between subchannels, thus avoiding repeated executions of the entire bit-loading algorithm.

A. Polite Identification of Bit-swaps

During SHOWTIME operation, the bit-swapping procedure swaps bits from one subchannel to another in order to maximize the minimum SINR margin across subchannels, thus keeping the line stable when the channel or noise slowly changes. Adaptation by weak users will typically not affect the strong users. On the other hand, strong users should be polite toward the weak users in their adaptation. If bit-swapping is performed among subchannels that are already used, then other users will not see an increase in crosstalk since the strong user does not increase its energy in any subchannel. However, if a bit is to be added to a subchannel that previously carried zero bits, then a rate-penalty may be caused to the weak user since it will experience more interference because of the extra energy used by the strong user on that subchannel. Polite addition of bits to new subchannels can be achieved by using the rate-penalty table. Using steps 6 to 8 of Algorithm 2, the strong user can determine the subchannel where the bits are to be added, while causing minimal penalty to the weak user.

B. Polite Identification of Gain-swaps and energy-increase

Fine gain (or energy) scaling, which changes the energy allocated to the subchannels, is typically used to aid bit-swapping and equalize the margin across subchannels[3]. While minor changes in the energies after initial bit-loading will not change the interference significantly, the near-far situation poses an interesting problem. During initial bit-loading, the strong user will typically back-off its total energy from the allowed maximum and will use more of the high frequency subchannels. If the noise of the strong user changes, then the margin may drop below the target value prompting the modem to swap bits and energies, and even increase the total energy. In current systems, bits would be moved to subchannels with the lowest incremental-energy so that the maximum margin

[3]An equal margin-per-subchannel policy is usually adopted since the performance of a multicarrier system depends on the bit-error-rate of the worst subchannel. Such equal margin imposition may be the choice of the SMC that is not dynamic in terms of the speed of line-condition changes.

TABLE III

SIMULATION PARAMETERS

No. of subchannels	256	Subchannel Width (KHz)	4.3125
Bit-cap	15	Coding Gain (dB)	3
Symbol Rate (KHz)	4	Max Power (dBmW)	20.4
Target Margin (dB)	6	PSD Mask (dBm/Hz)	-38.5
Uncoded Gap (dB)	9.8	Line Gauge	24

can be achieved with minimum energy-increase. However, this may not be polite towards weak users. Instead, strong users should try to adapt while causing minimum harm to weak users. With this constraint, the penalty table can be used to achieve politeness as follows:

Algorithm 3 Polite energy-increase by Strong User

1: Remove bits and energies from subchannels that are below the target margin
2: Execute steps 5 to 11 of Algorithm 2 until the target data rate is met
3: Implement appropriate bit/gain-swaps and energy-increase

VI. NUMERICAL RESULTS AND DISCUSSIONS

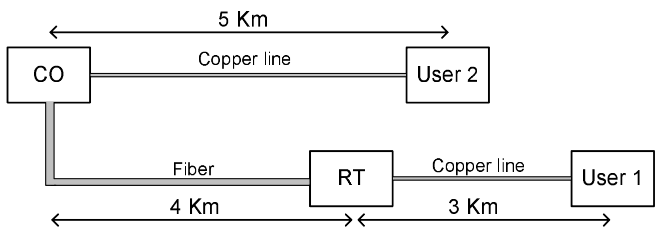

Fig. 2. Simulation Setup - ADSL CO/RT scenario

An ADSL downstream CO/RT scenario (Fig. 2) was simulated to evaluate the performance of SBLC. For comparison with the computationally complex OSB, only two users are considered. The simulation parameters are summarized in Table III. The noise is a combination of -140 dBm/Hz AWGN along with Noise A, which is a mixture of 16 ISDN, 4 HDSL, and 10 ADSL disturbers. The SBLC algorithm converged within 3 iterations among the users. A discrete-bit loading version of IWF was also simulated where the FM LC algorithm was executed by the RT user, and the RA LC algorithm was executed by the CO user until convergence.

Figure 3 compares the rate region of SBLC with those obtained by OSB and IWF. The rate region of SBLC is very close to that of the optimal OSB. The PSDs in Fig. 4 further verify the closeness. The stronger RT user fully utilizes the higher frequency band and then loads bits in the lower frequencies while causing minimum penalty to the weaker CO user. This enables SBLC to obtain huge gains over IWF and very close performance to OSB. Simulation results for situations with more than two users are presented in [18]. Specifically, a 25-user example in [18] illustrates the low-complexity and practicality of the SBLC algorithm.

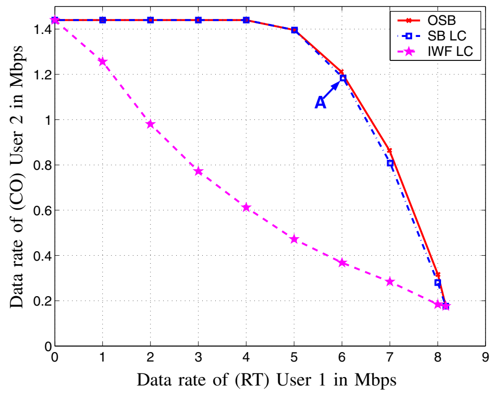

Fig. 3. Comparison of rate region achieved by SBLC with OSB and IWF

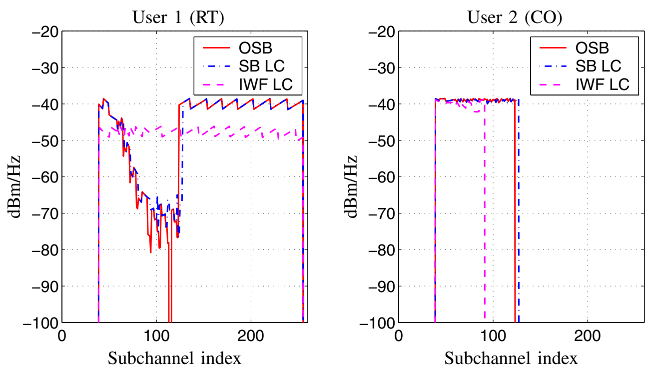

Fig. 4. Energy allocation of SBLC, OSB, and IWF when R_1=6 Mbps

To illustrate the performance of the polite energy-increase method, the operating point A in Fig. 3 is chosen. This operating point lies in the region where the crosstalk from the stronger RT user to the CO user is comparable to the noise experienced by the CO user. Hence, the operating point A represents the delicate balance between the crosstalk and noise that is achieved by the spectrum balancing algorithm and is an ideal point to evaluate the robustness of the adaptation algorithm. The initial margin for the two users is 6 dB. The noise of the stronger RT user is then increased by 10 dB in subchannels 101 through 140. In response, the RT user performs bit- and gain-swapping and increases its energy. Therefore, the CO user sees more interference and also tries to adapt. Fig. 5 shows the margin distribution of the CO user when the RT uses the incremental energy table to obtain minimal energy-increase, and also when Algorithm 3 is utilized. The left subplot shows the margin of the CO user before it performs bit-swapping. The RT's minimum-energy-increase method causes more subchannels of the CO user to have negative margin compared to the polite energy-increase method. Although, the RT's polite energy-increase method initially causes a lower minimum margin for the CO, a Reed-Solomon code will cope better with fewer subchannels having negative margin. Moreover, with sufficient speed of bit-swapping by the CO user, the margin degradation will be lower than shown in the figure. After bit-swapping is completed by the CO user, the margin distribution in the right subplot of Fig. 5 is obtained. Clearly, the polite energy-increase algorithm of the RT allows the CO user to achieve a minimum margin of 2.55 dB after bit-swapping, while the minimum-energy-increase method results in an undesirable minimum margin of -0.46 dB, which will usually cause the weaker CO user to re-initialize at a lower data-rate. This clearly illustrates the importance of polite adaptation by strong users, which is enabled by the framework of the SBLC algorithm.

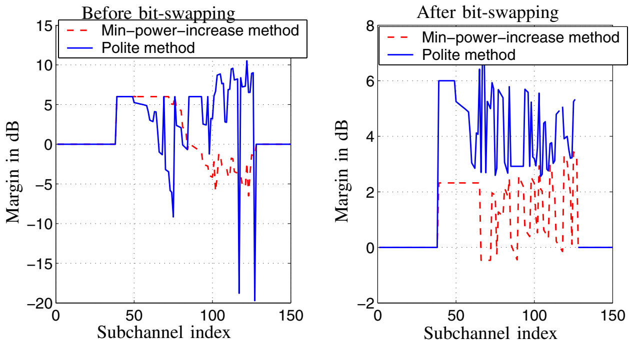

Fig. 5. Margin distribution of the CO user before and after it performs bit-swapping in response to changing interference from RT user

The better performance of the polite algorithm can be explained by the RT user's PSDs shown in Fig. 6. While the polite algorithm still avoids causing interference in the intermediate subchannels, the energy-efficient algorithm causes more interference in those subchannels, thereby affecting the performance of the CO user. It should be noted that the RT user's energy was 17.69 dBmW using the polite method and 18.55 dBmW using the minimum-energy-increase method. However, total energy alone does not determine the harm caused to the weaker user. The more important factor is the reduction of interference in the overlapping subchannels.

VII. Conclusions

A low-complexity discrete bit-loading algorithm called SBLC was presented for spectrum balancing in multi-user multicarrier systems. The SBLC algorithm can be easily implemented by using a rate-penalty table and extra checks along with the LC algorithm. The proposed algorithm could be used to either perform distributed bit-loading or efficiently determine the water-level scaling factors for band-preference. Simulation results demonstrated that near-optimal performance can be achieved by the SBLC algorithm. Moreover, procedures to determine bit- and gain-swaps were developed to ensure politeness toward the weaker users. Such adaptation capability to changes in the channel and noise conditions is a unique advantage of the proposed algorithm.

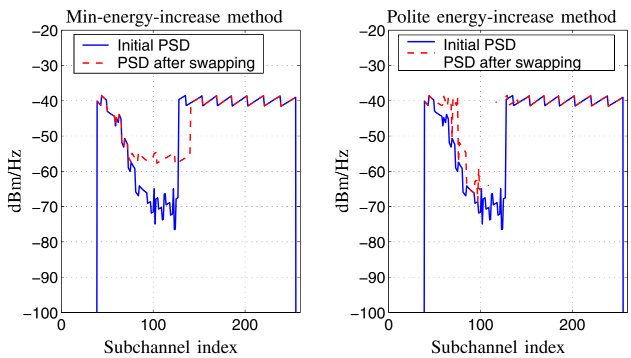

Fig. 6. RT user's PSDs obtained by two methods to determine bit/gain-swaps

Acknowledgements

This work was supported by the NSF under contract CNS-0427711 and France Télécom R&D. The authors thank Chan Soo Hwang and Moshe Malkin for helpful discussions.

References

[1] T. Starr, M. Sorbara, J. M. Cioffi, and P. J. Silverman, *DSL Advances*. Prentice Hall, 2003.

[2] Dynamic Spectrum Management Technical Report, "ATIS Committee NIPP Pre-published document ATIS-PP-0600007," May 2007.

[3] W. Yu, G. Ginis, and J. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," *IEEE J. Select. Areas Commun.*, vol. 20, no. 5, pp. 1105–1115, June 2002.

[4] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, May 2006.

[5] W. Yu and R. Lui, "Dual Methods for Nonconvex Spectrum Optimization of Multicarrier Systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, July 2006.

[6] R. Cendrillon and M. Moonen, "Iterative Spectrum Balancing for Digital Subscriber Lines," in *Proc. IEEE ICC*, vol. 3, May 2005, pp. 1937–1941.

[7] J. Papandriopoulos and J. S. Evans, "Low-Complexity Distributed Algorithms for Spectrum Balancing in Multi-User DSL Networks," in *Proc. IEEE ICC*, vol. 7, June 2006, pp. 3270–3275.

[8] D. Yu, K. Seong, and J. M. Cioffi, "Multiuser Discrete Bit-loading for Digital Subscriber Lines," in *Proc. IEEE ICC*, June 2007, pp. 2755–2760.

[9] J. M. Cioffi, W. Rhee, M. Mohseni, and M. H. Brady, "Band Preference in Dynamic Spectrum Management," in *Eurasip Conference on Signal Processing*, Sept. 2004.

[10] W. Lee, Y. Kim, M. Brady, and J. M. Cioffi, "Band Preference Dynamic Spectrum Management in a DSL environment," in *Proc. of IEEE Globecom*, Nov. 2006.

[11] ——, "Distributed Band-Preference Dynamic Spectrum Management for Digital Subscriber Lines," *in preparation*.

[12] R. Cendrillon, J. Huang, M. Chiang, and M. Moonen, "Autonomous Spectrum Balancing for Digital Subscriber Lines," *IEEE Trans. Signal Processing*, vol. 55, no. 8, pp. 4241–4257, Aug. 2007.

[13] H. E. Levin, "A complete and optimal data allocation method for practical discrete multitone systems," in *Proc. of IEEE Globecom*, vol. 1, Nov. 2001, pp. 369–374.

[14] J. Campello de Souza, *Discrete Bit Loading for Multicarrier Modulation Systems*. Ph.D. dissertation, Department of Electrical Engineering, Stanford University, Stanford, CA, 1999.

[15] J. M. Cioffi, *EE379C course reader*. Stanford University, 2005. [Online]. Available: http://www.stanford.edu/class/ee379c

[16] J. Lee, R. V. Sonalkar, and J. M. Cioffi, "Multiuser bit loading for multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1170–1174, July 2006.

[17] ——, "A multi-user power control algorithm for digital subscriber lines," *IEEE Commun. Lett.*, vol. 9, no. 3, pp. 193–195, Mar. 2005.

[18] S. Jagannathan and J. M. Cioffi, "Distributed adaptive bit-loading for spectrum optimization in multi-user multicarrier systems," *Physical Communication (2008), doi:10.1016/j.phycom.2008.01.005*.

[19] D. S. Taubman and M. W. Marcellin, *JPEG2000: Image Compression Fundamentals, Standards, and Practice*. Kluwer Academic Publishers, 2002.