Margin Optimization in Digital Subscriber Lines Employing Level-1 Dynamic Spectrum Management

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***Abstract*— This paper investigates the optimization of margins in a digital-subscriber-line (DSL) system employing level-1 dynamic spectrum management. A central spectrum management center determines the empirical statistics of the noise variance of each DSL line using long-term observations and optimizes the margin allocation using the statistics. Compared to current systems that allocate ad-hoc margin values, the presented algorithms improve the line's stability, throughput-guarantee, and politeness to other lines in the binder.**

I. Introduction

Digital-subscriber-lines (DSLs) are becoming a widespread means of providing broadband connections with over 200 million customers worldwide. To further improve the coverage of DSL, service providers are aiming for higher data rates and greater reach. In addition, new applications such as video, VoIP, etc. are imposing quality-of-service (QoS) requirements such as low delay and high stability. These challenges are being addressed with the introduction of dynamic spectrum management (DSM) methods [1][2].

The central component of a DSL system employing DSM is a spectrum management center (SMC), which monitors the channel and noise of a set of lines and optimizes their DSL parameters such as data-rate, power, margin, power-spectral-density (PSD) mask, forward-error-correction (FEC) parameters, etc. When the channel is static and the noise is stationary, optimization of the DSL parameters can be performed based on the channels and noise spectra at initialization. However, DSLs also experience time-varying noises, which arise from various sources and can have different characteristics. Intermittent noise such as impulse noise is a type of non-stationary noise, which causes burst errors in the data packets. The resulting short-duration error events are best handled using FEC codes such as Reed-Solomon codes [3]. Another type of time-varying noise, also encountered in DSLs, is the one for which the noise spectrum stays relatively stationary over a long duration of time (on the orders of hours), but then changes to a different noise spectrum. Such noise, referred to as quasi-stationary noise, is caused by varying crosstalk that occurs when other lines in the binder turn on/off, or when they adjust their PSDs. Noises from external devices such as TV, microwave, etc. also contribute to the quasi-stationary noise.

It is important for DSLs to be robust against these non-periodic, quasi-stationary noises to provide a reliable service. However, a comprehensive solution for dealing with such noises does not seem to be available in literature. The margin parameter, also known as the signal-to-interference-and-noise-ratio (SINR) margin, is available for use by service providers to make a DSL robust to unexpected changes in the channel or noise conditions. Such a parameter is also likely to be used in other wireline multicarrier systems such as power-line-communication (PLC) networks. While formal methods exist to optimize most parameters of a DSL service, the value for the margin parameter has been chosen in a rather ad-hoc manner, such as 6 dB, which might not be sufficient or might be too much for the intended purpose.

To quantify and to provide sufficient robustness against quasi-stationary noise, this paper considers the optimization of the margin along with the energy allocation in a level-1 DSM setting, where each line is individually optimized using the knowledge of its own channel and noise conditions. The concept of *probability-of-outage* of a DSL is introduced to quantify its stability at a given data rate and target probability-of-error using not only the noise condition at initialization but also the empirical noise variance distribution, which can be acquired by monitoring the line over a long time. Then, this paper presents algorithms to optimize both the margin and spectrum using the probability-of-outage concept.

II. System Model

A DSL system is considered as shown in Fig. 1 where a central office (CO) or a remote terminal (RT) (or a combination) serves users using copper-lines that share the same binder. Let the total number of lines in the binder be K. A subset of U lines, $\mathcal{U} \subseteq \{1, 2, \ldots, K\}$, called the *managed lines*, is monitored by an SMC, while the remaining $K - U$ lines, which may be served by other service providers in an unbundled environment, are called the *unmanaged lines*.

The managed DSL lines employ discrete multi-tone transmission where an inter-symbol-interference (ISI) channel has been converted into N orthogonal tones in the frequency domain. The tones are sufficiently narrow so that the channel-magnitude response appears flat. Line u's direct channel on tone n is denoted by $H^n_{u,u}$, while the far-end-crosstalk transfer function from line v to line u is denoted by $H^n_{u,v}$. The lines are assumed to be synchronized, and free from inter-carrier-interference and ISI.

The variance of the noise, including the crosstalk from the other managed lines, experienced by line u on tone n is denoted by $(\tilde{\sigma}^n_u)^2$. The per-tone noise variance in dBm is

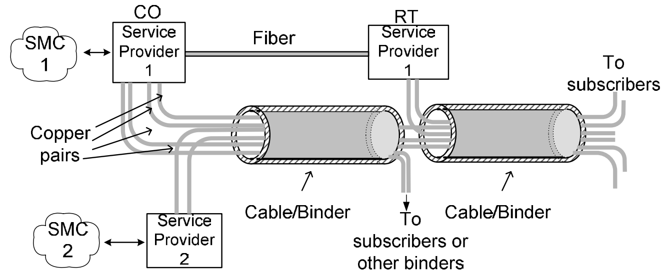

Fig. 1. A typical DSL system where the lines of various service providers are monitored by their SMCs.

modeled by a random process, $\tilde{X}_u^n(t)$, whose marginal distribution is empirically obtained by the SMC using long-term observations of each line. Since noise sources as well as the unmanaged crosstalk (from other $K-U$ DSLs) usually affect a band of frequencies, the variation in the noise-variance across tones is expected to be correlated. Such correlation needs to be adequately captured in the noise-variance distribution model to aid the optimization of the margins, and is described in Section VI.

III. The Concept of Margin in Wire-line Communications

The role of margin in communication systems is succinctly explained by the following statements from [4]: "The margin specification is a way of stating that there must be an ability for the technology to withstand impairments that are not normally anticipated. These excessive impairments can be caused by temperature variations, unexpected noise influence, etc." Traditionally, the margin has been used to determine how much increase in the noise and interference (from other lines in the binder) can be withstood by the line until the probability of error increases above the allowed maximum value[1].

A. SINR margin

The margin is usually applied as a reduction factor in the SINR. If $\mathcal{E}_u^n$ is the energy allocated to user u on tone n, then $\text{SINR}_u^n = \frac{\mathcal{E}_u^n |H_{u,u}^n|^2}{(\tilde{\sigma}_u^n)^2}$, and the number of bits is

$$b_u^n = \log_2\left(1 + \frac{\text{SINR}_u^n}{\tilde{\gamma}_u \Gamma}\right), \tag{1}$$

where $\tilde{\gamma}_u$ is the *SINR margin* and Γ is the SNR gap of the code [5]. Equation (1) provides the number of bits on tone n that can be sustained at the desired maximum probability-of-error (implicitly specified by Γ) if the SINR decreases by at most $\tilde{\gamma}_u$. Since DSL channels are usually static, noise-spectrum variations are the focus of this paper. A higher SINR margin provides more protection against larger noise increases.

Many DSL provisioning systems monitor a line over a long time and provide the service if the line operates satisfactorily at a certain data rate using an ad-hoc margin value such as 6 dB [4][6]. However, the line may end up with more margin than is actually required since the provisioning is based on the worst-case noise condition. Although a higher margin may protect the line from a larger noise change, it requires a larger transmit power, which causes more interference to other lines in the binder. If the maximum power is already being used, a higher margin would reduce the data rate of the line. Hence, it is desirable to restrict the SINR margin value to provide sufficient robustness against noise at the target data-rate while also being polite to other lines in the binder. Fortunately, a more aggressive provisioning of lines can be adopted when long-term observations of the noise are available. First, it is necessary to determine a metric to quantify the robustness of a DSL service in order to formulate the optimization problems. Such a metric is defined in the following section.

IV. Probability of Outage

A stable data-rate that can be guaranteed with a high probability is important for providing QoS in a broadband communication system. This requirement is captured by:

Definition 1: The *probability of outage*[2], $P_{out,u}$, is the probability that the line u can only operate below the target data rate when forced to meet the target probability of error.

The probability of outage is a very useful parameter in determining the QoS of a line. For example, a service provider may require that a line should be able to operate at 30 Mbps for 99% of the time with a bit-error-rate of 10^{-7} to provide a HDTV service. Such a requirement can be translated into a probability-of-outage requirement.

The target probability-of-error of a line is determined by the effective SINR gap, which is the sum in dB of Γ and $\tilde{\gamma}_u$. If the SINR margin is less than 0 dB, then the effective gap of the system is smaller than the gap required for the target error-rate. Hence, the target probability of error is no longer achievable at the target data rate, which means the line is in *outage*. Consequently, the line would either have to re-initialize to a lower data rate[3] or would have to reduce its data rate using mechanisms such as the seamless-rate-adaptation (SRA) procedure in VDSL. With this observation, the definition of the probability of outage translates to the probability that the SINR margin drops below 0 dB.

Let $\tilde{\gamma}_u^{\text{init}}$ be the SINR margin that is allocated to the line at initialization. A higher value of $\tilde{\gamma}_u^{\text{init}}$ will allow a larger noise-increase to be withstood before the margin drops below 0 dB. This observation is summarized in the following proposition:

Proposition 1: $P_{out,u}$ is a decreasing function of $\tilde{\gamma}_u^{\text{init}}$.

As $\tilde{\gamma}_u^{\text{init}}$ increases, the data rate of the line reduces. Therefore, the margins of the lines at initialization can be optimized to achieve the best trade-off between maximizing the data rates and improving the stability of the lines. Determining $P_{out,u}$ as a function of $\tilde{\gamma}_u^{\text{init}}$ is important to formulate and solve the margin optimization problems and is considered next.

[1] Any channel fluctuations caused by temperature or soil variations can also be countered by the SINR margin, and are easily incorporated into the algorithms presented in this paper.

[2] An outage event as defined in this paper is sometimes referred to as a *false positive* by service providers.

[3] A mechanism that restores the target data-rate when the noise condition improves is assumed to exist.

A. Determining the probability of outage

The probability of outage, $P_{out,u}$, as a function of $\tilde{\gamma}_u^{\text{init}}$ is determined by considering the per-tone margins as random variables, $\tilde{\Upsilon}_u^n$. Maintaining the PSD after initialization and before bit-swapping, the variation in the per-tone SINR margins caused by variations in the alien-noise spectrum is given by

$$\tilde{\Upsilon}_u^n(\text{dB}) = \tilde{\gamma}_u^{\text{init}}(\text{dB}) + (\tilde{\sigma}_u^{n,\text{init}})^2(\text{dBm}) - \tilde{X}_u^n(\text{dBm}), \qquad (2)$$

where $(\tilde{\sigma}_u^{n,\text{init}})^2$ is the per-tone noise-variance at initialization and $\tilde{X}_u^n$ is the random variable representing the current noise-variance on tone n for line u. Although the margin, $\tilde{\gamma}_u^{\text{init}}$, is allocated equally across tones during initialization, the per-tone margins can vary when the noise spectrum changes. Incorporating the per-tone margins into the outage probability depends on the adaptation functionality of the system.

1) Bit-swapping is used: In practice, many DSL systems implement a bit-swapping procedure that equalizes the margin across tones by moving bits from tones with higher energy cost to those with lower energy cost [5], thus maximizing the minimum SINR margin across all used tones.

When bit-swapping is used, the random variable $\tilde{\Upsilon}_u$ representing the equivalent SINR margin that is equal across tones needs to be obtained to determine the probability of outage. First the *equivalent SINR margin*, $\tilde{\gamma}_u$, corresponding to fixed per-tone SINR margins, $\tilde{\gamma}_u^n$, is defined as:

Definition 2: For fixed $\mathcal{E}_u^n$, $H_{u,u}^n$, $(\tilde{\sigma}_u^n)^2$, and $\tilde{\gamma}_u^n$ $\forall u, n$, let $b_u^n = \log_2\left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u^n \Gamma}\right)$, where $g_u^n = \frac{|H_{u,u}^n|^2}{(\tilde{\sigma}_u^n)^2}$. The *equivalent SINR margin* is defined to be the maximum value of $\tilde{\gamma}_u$ such that $b'^n_u = \log_2\left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u \Gamma}\right)$ and $\sum_n b'^n_u = \sum_n b_u^n = R_u$. In other words, for a given channel, noise spectrum, and PSD, the *equivalent SINR margin* is the SINR-margin, $\tilde{\gamma}_u$, that can be applied equally to all tones, while maintaining the data rate obtained using the unequal per-tone SINR margins, $\tilde{\gamma}_u^n$.

Consider the function $f(\tilde{\gamma}_u) = \sum_n \log_2\left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u \Gamma}\right) - R_u$. Since $f(\tilde{\gamma}_u)$ is a decreasing and continuous function of $\tilde{\gamma}_u$, bisection can be used to solve for the value of $\tilde{\gamma}_u$ that results in $f(\tilde{\gamma}_u) = 0$. However, a closed-form expression is desired to determine the probability of outage. Therefore, an approximate closed-form expression for the equivalent SINR margin is derived. The condition $\sum_n b'^n_u = \sum_n b_u^n = R_u$ implies

$$\prod_n \left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u^n}\right) = \prod_n \left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u}\right). \qquad (3)$$

Neglecting the addition of 1 on both sides of (3), we obtain $\tilde{\gamma}_u \approx \left(\prod_{n \in \mathcal{N}_{\text{used}}} \tilde{\gamma}_u^n\right)^{\frac{1}{N^*}}$ in linear scale, or in the log scale

$$\tilde{\gamma}_u(\text{dB}) \approx \frac{1}{N^*} \sum_{n \in \mathcal{N}_{\text{used}}} \tilde{\gamma}_u^n(\text{dB}), \qquad (4)$$

where $\mathcal{N}_{\text{used}} \subseteq \{1, \ldots, N\}$, (of size N^*), is the set of tones to which energy allocated during the bit-loading process, i.e, $\mathcal{E}_u^n \neq 0 \ \forall \ n \in \mathcal{N}_{\text{used}}$.

The approximation is accurate when the SINR is high, which is typical in DSL systems. In fact, exhaustive simulations indicated that even for low SINRs, the approximation for the equivalent SINR margin is within 0.5 dB of the actual value (and often very accurate) when the maximum deviation in the per-tone margins, $\tilde{\gamma}_u^n$, is less than 40 dB. This range is sufficient for most practical systems. Hence, the approximation can be used with a high degree of confidence.

Applying (4) to $\tilde{\Upsilon}_u^n$, the equivalent SINR margin random variable is obtained as $\tilde{\Upsilon}_u(\text{dB}) = \frac{1}{N^*} \sum_{n \in \mathcal{N}_{\text{used}}} \tilde{\Upsilon}_u^n(\text{dB})$. Using (2), the probability of outage is derived as:

$$P_{out,u} = Pr\Big\{\tilde{\Upsilon}_u(\text{dB}) < 0\Big\} = Pr\Big\{\frac{1}{N^*} \sum_{n \in \mathcal{N}_{\text{used}}} \tilde{\Upsilon}_u^n(\text{dB}) < 0\Big\}$$
$$= Pr\Big\{\frac{1}{N^*} \sum_{n \in \mathcal{N}_{\text{used}}} \tilde{X}_u^n(\text{dBm}) > \tilde{\gamma}_u^{\text{init}}(\text{dB}) +$$
$$\frac{1}{N^*} \sum_{n \in \mathcal{N}_{\text{used}}} (\tilde{\sigma}_u^{n,\text{init}})^2(\text{dBm})\Big\}. \qquad (5)$$

Therefore, the cumulative distribution function (cdf) of the random variable $\hat{X}_u = \frac{1}{N^*} \sum_{n \in \mathcal{N}_{\text{used}}} \tilde{X}_u^n$ is required to determine the probability of outage. Equation (5) shows that the outage probability of a line is a function of its initialization margin and noise spectrum. The metric captures the intuition that for a desired outage probability, the margin allocation can be reduced if the current noise is larger than its nominal value. The SMC can obtain the average of the noise-variances across tones at various instances of time and store the parameters of the empirical distribution such as its mean and variance. Since the set of used tones $\mathcal{N}_{\text{used}}$ may vary under different noise scenarios, the SMC can obtain the distributions for a few different combinations of used tones. This step requires little complexity since the DSL channel structure usually results in an almost fixed subset of used tones in a variety of noise scenarios with only a few tones being excluded when the noise is severe.

2) Bit-swapping is not used: If bit-swapping is not used, then the probability of outage will be determined by the minimum margin across tones, i.e.,

$$P_{out,u} = 1 - Pr\left\{\min_{n \in \mathcal{N}_{\text{used}}} \Big(\tilde{\Upsilon}_u^n(\text{dB})\Big) > 0\right\} =$$
$$1 - Pr\left\{\max_{n \in \mathcal{N}_{\text{used}}} \Big(\tilde{X}_u^n(\text{dBm}) - (\tilde{\sigma}_u^{n,\text{init}})^2(\text{dBm})\Big) < \tilde{\gamma}_u^{\text{init}}(\text{dB})\right\} \qquad (6)$$

using (2). In this scenario, the SMC should gather the statistics of the maximum noise deviation across the tones, i.e., $X_{\text{dev},u}(\text{dBm}) = \max_{n \in \mathcal{N}_{\text{used}}} \Big(\tilde{X}_u^n(\text{dBm}) - (\tilde{\sigma}_u^{n,\text{init}})^2(\text{dBm})\Big)$. The rest of the paper assumes that bit-swapping is used.

V. Optimization of SINR Margin in Level-1 DSM

In level-1 DSM, the SINR margin of a line can be optimized to provide better protection against uncertainties in its noise and interference. The SMC determines the initialization-margin using both the current noise spectrum and the distribution of the noise-variance, instead of allocating an ad-hoc margin value. Thus, the margin is allocated considering the long-term stability and throughput-guarantee of the service.

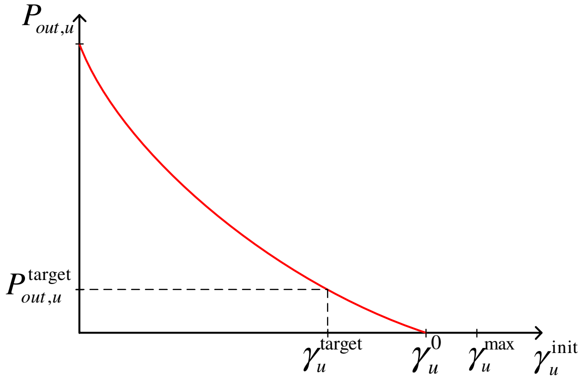

Fig. 2. Probability of outage as a function of the margin

When it is desired to provide a DSL service at a fixed data rate, e.g. HDTV, then the margin allocation can be used to minimize the outage probability. If the resulting outage probability meets the QoS requirement for the service, then the line can be provisioned at the target data-rate. This approach is considered by the following optimization problem:

$$\begin{array}{ll} \text{minimize} & P_{out,u} \\ \text{subject to} & \sum_{n=1}^{N} \log_2\left(1+\frac{\mathcal{E}_u^n |H_{u,u}^n|^2}{\Gamma\tilde{\gamma}_u^{\text{init}}(\tilde{\sigma}_u^{n,\text{init}})^2}\right) = R_u^{\text{target}} \\ & \sum_{n=1}^{N} \mathcal{E}_u^n \le \mathcal{E}_u^{\text{tot}} \\ & \mathcal{E}_u^n \ge 0,\ \forall n \\ & \tilde{\gamma}_u^{\text{init}} \ge 1 \\ & \tilde{\gamma}_u^{\text{init}} \le \tilde{\gamma}_u^{\max} \end{array}, \quad (7)$$

where the objective is to minimize the outage probability, which is a function of the initialization margin and noise spectrum. The optimization variables are the SINR margin, $\tilde{\gamma}_u^{\text{init}}$, and the energy allocation, $\mathcal{E}_u^n$, which are selected to satisfy the constraints on the target-data-rate and the sum-energy across tones as well as to meet the requirement that the SINR margin should be at least 0 dB (i.e., 1 in linear scale). A limit on the maximum SINR margin, $\tilde{\gamma}_u^{\max}$, is also imposed to ensure politeness to other lines by not allowing the line to transmit too much power. Constraints on the maximum number of bits on a tone (bitcap) and PSD mask can also be easily incorporated.

Proposition 1 implies that minimizing the probability of outage corresponds to maximizing the margin. Therefore, if the minimum outage probability in (7) is non-zero, then the margin should be set to the maximum possible value. One obvious restriction is that $\tilde{\gamma}_u^{\text{init}} \le \tilde{\gamma}_u^{\max}$. However, the target data-rate and energy constraints also play a role. The maximum margin, $\tilde{\gamma}_u^{\text{en}} > 0$, subject to the data-rate and energy constraints alone is provided by the margin-adaptive water-filling (MA-WF) algorithm [5]. Therefore, $\tilde{\gamma}_u^{\text{init}}$ must be set to $\tilde{\gamma}_u^{\min} = \min(\tilde{\gamma}_u^{\text{en}}, \tilde{\gamma}_u^{\max})$ when $P_{out,u}$ is non-zero as shown in Fig. 2. However, if the minimum outage probability is zero, then the line could be more polite by choosing the lowest margin possible, $\tilde{\gamma}_u^0$, while maintaining the target data rate.

Combining the two cases, the optimal value of $\tilde{\gamma}_u^{\text{init}}$ is $\min(\tilde{\gamma}_u^0, \tilde{\gamma}_u^{\min})$. Thus, the margin of the line is optimized to provide the target data rate, while minimizing the probability of outage.

An alternate approach to service provisioning is to maximize a line's data rate, while meeting an outage probability requirement. The corresponding optimization problem is:

$$\begin{array}{ll} \text{maximize} & \sum_{n=1}^{N} \log_2\left(1+\frac{\mathcal{E}_u^n |H_{u,u}^n|^2}{\Gamma\tilde{\gamma}_u^{\text{init}}(\tilde{\sigma}_u^{n,\text{init}})^2}\right) \\ \text{subject to} & \sum_{n=1}^{N} \mathcal{E}_u^n \le \mathcal{E}_u^{\text{tot}} \\ & \mathcal{E}_u^n \ge 0,\ \forall n \\ & \tilde{\gamma}_u^{\text{init}} \ge 1 \\ & P_{out,u} \le P_{out,u}^{\text{target}} \end{array}, \quad (8)$$

where the objective is to maximize the data rate. The optimization variables are $\tilde{\gamma}_u^{\text{init}}$ and $\mathcal{E}_u^n$, which are selected subject to constraints on the maximum sum-energy and outage probability, with the minimum SINR margin being 0 dB.

Problem 8 is simplified by first determining the minimum margin, $\tilde{\gamma}_u^{\text{target}}$, that is required to satisfy the target probability of outage. Then, the optimal value of $\tilde{\gamma}_u^{\text{init}}$ is simply $\max(1, \tilde{\gamma}_u^{\text{target}})$ since the data rate is a strictly decreasing function of the margin. Finally, the rate-adaptive water-filling (RA-WF) algorithm is executed using the optimal SINR margin to maximize the data rate of the line.

One caveat in the solutions to problems (7) and (8) is that determining the outage probability as a function of the initialization margin requires knowing the set of loaded tones, $\mathcal{N}_{\text{used}}$, in advance. To tackle this issue, iterations can be performed with an appropriate starting configuration for the set of loaded tones until the solution converges. One reasonable starting point is to assume a 0 dB margin and execute the RA-WF algorithm to obtain the largest possible set of loaded tones, $\mathcal{N}_{\text{used}}^{\max}$. For the problem in (8), $\tilde{\gamma}_u^{\text{target}}$ can first be determined using $\mathcal{N}_{\text{used}} = \mathcal{N}_{\text{used}}^{\max}$. RA-WF can then be executed using this margin, followed by refining $\tilde{\gamma}_u^{\text{target}}$ using the new set of loaded tones. The procedure continues until the set of loaded tones, $\mathcal{N}_{\text{used}}$, converges. For the problem in (7), if the outage probability using $\tilde{\gamma}_u^{\min}$ is zero, then $\tilde{\gamma}_u^0$ is initially determined using $\mathcal{N}_{\text{used}} = \mathcal{N}_{\text{used}}^{\max}$. Then the fixed-margin water-filling algorithm (FM-WF), which minimizes the total energy at a given data-rate and margin is executed. As before, $\tilde{\gamma}_u^0$ can be refined using the new set of used tones and the FM-WF is iteratively executed until the set of used tones $\mathcal{N}_{\text{used}}$ converges. The resulting algorithms for the two problems are summarized in Algorithms 1 and 2. While a convergence proof is not available, simulations showed convergence under a wide variety of channel and noise conditions. After solving (7) or (8), the SMC sends the optimal SINR margin to the modem, which then performs bit-loading in the usual way. Thus, the distributed nature of DSL operation is preserved, with only an extra margin optimization step being performed by the SMC.

The next section presents a model for the marginal distribution of the noise-variance for illustration of the simulation results and possibly for easy implementation in practice.

VI. Noise-variance Distribution Model

The noise variance on each tone is assumed to follow a Beta distribution. This distribution is chosen since its flexibility can

Algorithm 1 Minimizing the Outage Probability - Prob. (7)

Obtain $\tilde{\gamma}_u^{\text{en}}$ using MA-WF
Calculate $P_{out,u}$ using $\tilde{\gamma}_u^{\text{min}} = \min(\tilde{\gamma}_u^{\text{en}}, \tilde{\gamma}_u^{\text{max}})$
if $P_{out,u} \neq 0$ **then**
    $\tilde{\gamma}_u^{\text{init,opt}} = \tilde{\gamma}_u^{\text{min}}$
else
    Execute RA-WF with 0 dB margin and obtain the set of used tones, $\mathcal{N}_{\text{used}}^{\text{max}}$. Set $\mathcal{N}_{\text{used}} = \mathcal{N}_{\text{used}}^{\text{max}}$.
    repeat
        Determine $\tilde{\gamma}_u^0$ such that $P_{out,u} = 0$ using the noise-variance statistics corresponding to $\mathcal{N}_{\text{used}}$
        Execute FM-WF with margin $\tilde{\gamma}_u^0$ and data-rate R_u^{target} and obtain the new set of used tones $\mathcal{N}_{\text{used}}$
    until $\mathcal{N}_{\text{used}}$ converges
    $\tilde{\gamma}_u^{\text{init,opt}} = \min\left(\tilde{\gamma}_u^0, \tilde{\gamma}_u^{\text{min}}\right)$

Algorithm 2 Maximizing the Data Rate - Prob. (8)

Execute RA-WF with 0 dB margin and obtain the set of used tones, $\mathcal{N}_{\text{used}}^{\text{max}}$. Set $\mathcal{N}_{\text{used}} = \mathcal{N}_{\text{used}}^{\text{max}}$.
repeat
    Determine $\tilde{\gamma}_u^{\text{target}}$ such that $P_{out,u} = P_{out,u}^{\text{target}}$ using the noise-variance statistics corresponding to $\mathcal{N}_{\text{used}}$
    Execute RA-WF with margin $\tilde{\gamma}_u^{\text{target}}$ and obtain the new set of used tones $\mathcal{N}_{\text{used}}$
until $\mathcal{N}_{\text{used}}$ converges
$\tilde{\gamma}_u^{\text{init,opt}} = \max\left(1, \tilde{\gamma}_u^{\text{target}}\right)$

allow a good fit with the observed noise variances. Specifically, for line u on tone n, the random variable representing the noise variance in dBm, $\tilde{X}_u^n$, is expressed as

$$\tilde{X}_u^n = \tilde{M}_u^n + \tilde{D}_u^n \tilde{Y}_u^n, \tag{9}$$

where the constants $\tilde{M}_u^n$ and $(\tilde{M}_u^n + \tilde{D}_u^n)$, respectively, are the minimum and maximum values, in dBm, of line u's noise variance on tone n as illustrated in Fig. 3. $\tilde{Y}_u^n$ is a Beta random variable, which is characterized by its *shape* parameters a_u^n and b_u^n, or equivalently by its mean, $\mu_{\tilde{Y}_u^n}$, and standard deviation, $\sigma_{\tilde{Y}_u^n}$. The SMC fits the observed noise variances on each tone to determine $\tilde{M}_u^n$, $\tilde{D}_u^n$, a_u^n, and b_u^n.

A first-order model is employed to capture the noise correlation across tones. Assume that the correlation coefficient between the noise-variance on adjacent tones is α, i.e., $\rho_{\tilde{X}_u^n \tilde{X}_u^{n+1}} = \alpha$ for $n = 1, \ldots, N-1$. Define the normalized random variable $\hat{Y}_u^n = \frac{\tilde{Y}_u^n - \mu_{\tilde{Y}_u^n}}{\sigma_{\tilde{Y}_u^n}}$. Then $\mathbb{E}(\hat{Y}_u^n \hat{Y}_u^{n+1}) = \rho_{\tilde{X}_u^n \tilde{X}_u^{n+1}} = \alpha$. To generate the correlated random variables, the following procedure is adopted:

$$\begin{aligned} \hat{Y}_u^1 &= \hat{Z}_u^1 \\ \hat{Y}_u^n &= \alpha \hat{Y}_u^{n-1} + \sqrt{1-\alpha^2}\hat{Z}_u^n, \ 1 < n \leq N, \end{aligned} \tag{10}$$

where $\hat{Z}_u^n$ are independent random variables with zero mean and unit variance. The model has a physical interpretation that adjacent tones are more correlated and the correlation across tones drops exponentially as the tone-separation increases.

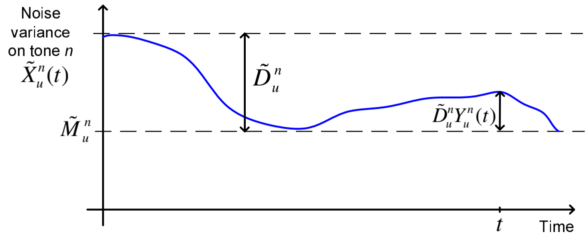

Fig. 3. Illustration of the Beta distribution model for the noise-variance.

A linear combination of two Beta random variables is well approximated by another Beta random variable [7]. Hence, $\hat{Z}_u^n$ is approximately Beta after appropriate scaling and shifting. Using (10), each $\hat{Y}_u^n$ is recursively obtained as a linear combination of the $\hat{Z}_u^n$'s as

$$\begin{aligned} \hat{Y}_u^1 &= \hat{Z}_u^1 \\ \hat{Y}_u^n &= \sqrt{1-\alpha^2}\left(\sum_{j=2}^n \alpha^{n-j}\hat{Z}_u^j\right) + \alpha^{n-1}\hat{Z}_u^1, \ 1 < n \leq N. \end{aligned} \tag{11}$$

Using (9) and (11), the random variable $\hat{X}_u$ from Section IV-A.1 is then expressed as:

$$\begin{aligned} \hat{X}_u = &\sum_{n \in \mathcal{N}_{\text{used}}} \frac{\left(\tilde{M}_u^n + \tilde{D}_u^n \mu_{\tilde{Y}_u^n}\right)}{N^*} + \frac{1}{N^*}\sum_{n \in \mathcal{N}_{\text{used}}} \tilde{D}_u^n \sigma_{\tilde{Y}_u^n} \alpha^{n-1}\hat{Z}_u^1 \\ &+ \sqrt{1-\alpha^2}\sum_{j=2}^N \hat{Z}_u^j \left(\frac{1}{N^*}\sum_{n \in \mathcal{N}_{\text{used}} \setminus \{1,\ldots,j-1\}} \tilde{D}_u^n \sigma_{\tilde{Y}_u^n}\alpha^{n-j}\right). \end{aligned} \tag{12}$$

Since (12) is a linear combination of independent Beta random variables, which have finite mean, variance and third central moments, and typically satisfy the Lyapunov condition [8] for various shape parameters, $\hat{X}_u$ converges to a normal distribution using the Lyapunov central limit theorem [8]. The mean, $\mu_{\hat{X}_u}$, and standard deviation, $\sigma_{\hat{X}_u}$, of $\hat{X}_u$ are then

$$\begin{aligned} \mu_{\hat{X}_u} &= \frac{1}{N^*}\sum_{n \in \mathcal{N}_{\text{used}}}\left(\tilde{M}_u^n + \tilde{D}_u^n \mu_{\tilde{Y}_u^n}\right) \\ \sigma_{\hat{X}_u} &= \left(\frac{1}{N^*}\sum_{n \in \mathcal{N}_{\text{used}}} \tilde{D}_u^n \sigma_{\tilde{Y}_u^n}\alpha^{n-1}\right)^2 + \\ &\quad (1-\alpha^2)\sum_{j=2}^N \left(\frac{1}{N^*}\sum_{n \in \mathcal{N}_{\text{used}} \setminus \{1,\ldots,j-1\}} \tilde{D}_u^n \sigma_{\tilde{Y}_u^n}\alpha^{n-j}\right)^2. \end{aligned}$$

Let $\tilde{\sigma}_{avg,u}^{\text{init}} = \frac{1}{N^*}\sum_{n \in \mathcal{N}_{\text{used}}} \tilde{\sigma}_u^{n,\text{init}}$. Finally, using (5) $P_{out,u}$ is expressed as:

$$P_{out,u} = Q\left(\frac{\tilde{\gamma}_u^{\text{init}} + \tilde{\sigma}_{avg,u}^{\text{init}} - \mu_{\hat{X}_u}}{\sigma_{\hat{X}_u}}\right). \tag{13}$$

VII. Numerical Results

This section presents numerical results to illustrate the use of the proposed margin optimization algorithms. A 24-gauge, 4 km ADSL loop is considered, and a 3 dB coding gain and 9.8 dB uncoded gap were used. The Beta distribution model was used to model the variation in the noise-spectrum. The minimum noise spectrum was chosen to be a flat, -140 dBm/Hz AWGN and the maximum noise spectrum was chosen to be an ANSI Noise A PSD [9], which is a mixture of 16 ISDN, 4 HDSL, and 10 ADSL disturbers, with a modified noise floor of -119 dBm/Hz. The shape parameters of the Beta distributions were chosen to be $a_u^n = b_u^n = 4$ and the correlation coefficient of the noise-variance between adjacent tones was chosen to be 0.9. This value is reasonable since the noise powers of adjacent tones are expected to be highly correlated. Three initialization-noise-spectra corresponding to the minimum noise level, close-to-maximum noise level, and an intermediate noise level (equal to the average of the maximum and minimum) were chosen.

For each of the noise-spectra, the problem in (7) was solved with different target data-rates[4]. The resulting trade-off between stability (quantified by the outage probability) and target data-rate is shown in Fig. 4. Such a plot can be used by service providers for determining the service for each line in the network. In the current example, an outage probability of 1% can be achieved by provisioning the line with at most 3.2, 4.16 and 4.22 Mbps, respectively, for the close-to-maximum, minimum, and intermediate noises. From Fig. 5, the corresponding optimal SINR margins are 6.85 dB and 18.58 dB for the intermediate and minimum noise levels. For the close-to-maximum noise level case, the outage probability does not exceed 1% even when the margin is 0 dB, which is therefore the optimal solution. The figures also indicate the situation when a 6 dB margin is used without considering the initialization-noise level and the noise distribution. For the intermediate noise-level, the outage probability using 6 dB margin is 2×10^{-2}, which is not very far from the desired value. However, for the close-to-maximum noise case, the 6 dB margin unnecessarily causes a loss in data-rate of 1.1 Mbps (by operating at $P_{out,u} = 2.5 \times 10^{-7}$, which is well below the target), while for the minimum noise case, the operating point is unstable since the outage-probability is close to 1 although the data-rate is very high. By using the margin optimization algorithm, such situations where too much excess margin is allocated could be avoided, thereby reducing the crosstalk to other lines or improving the data-rate. In addition, unstable situations, where a line re-initializes often because the provisioned data rate is too high, can be avoided.

VIII. Conclusions

Algorithms to optimize the SINR margins in DSL systems employing level-1 DSM have been developed to provide a high degree of stability, while maintaining politeness within the binder. Long-term throughput and stability guarantees can be provided for the lines when the system experiences quasi-stationary noise by allocating the margins while taking into account both the noise spectrum at initialization as well as the long-term noise-variance distribution. The methodology developed in this paper enables more aggressive provisioning of DSL services, which will result in increased coverage of DSL and better QoS for upcoming video and VoIP applications.

[4]Equivalently, the plot could have been obtained by solving the problem in (8) with different values for the desired outage probabilities.

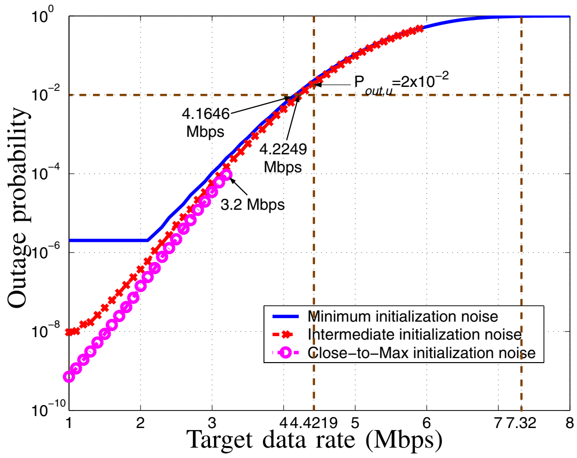

Fig. 4. Optimal outage probability vs target data-rate in a 4 km ADSL loop.

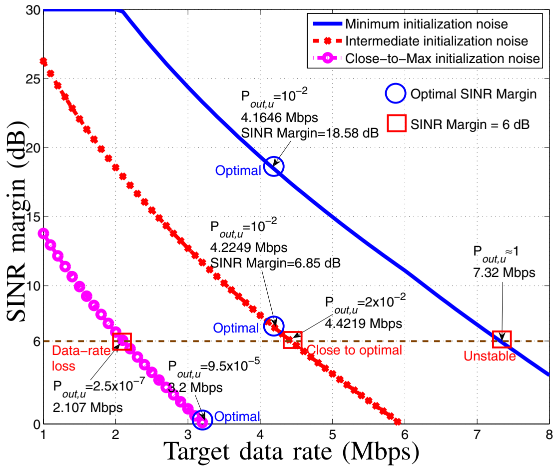

Fig. 5. Optimal margins required corresponding to Fig. 4.

Acknowledgements

This work was supported by the NSF under contract CNS-0427711 and France Télécom R&D.

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