Margin Optimization in Digital Subscriber Lines Employing Level-2 Dynamic Spectrum Management

Sumanth Jagannathan, Chan Soo Hwang, John M. Cioffi
Department of Electrical Engineering, Stanford University, Stanford, CA 94305
Email: {sumanthj, cioffi}@stanford.edu, cshwang@ieee.org

***Abstract*— This paper investigates the optimization of margins in digital subscriber lines employing level-2 dynamic spectrum management, where lines are jointly optimized with an emphasis on their stability, throughput-guarantee, and politeness. A new definition of *noise margin* and the concept of *probability of outage* are introduced, which allow for an easier formulation of the problems of joint margin and spectra optimization of the lines taking into account their stabilities. The solutions to the highly-coupled optimization problems are simplified using *noise margins* instead of signal-to-interference-and-noise-ratio margins. The presented algorithms provide up to 100% improvements in the data-rates or a reduction in the probability of outage by an order of magnitude, thus improving the stability of the lines at high data rates.**

I. Introduction

Digital-subscriber-lines (DSLs) are becoming a widespread method of providing broadband connections with over 200 million customers worldwide. The challenge of improving the coverage of DSL via higher data rates and greater reach, and supporting new applications, such as video, VoIP, etc., which impose quality-of-service (QoS) requirements such as low delay and high stability is being addressed with the introduction of dynamic spectrum management (DSM) methods [1][2].

In level-2 DSM, a spectrum management center (SMC) can use the knowledge of the direct and crosstalk channels of the lines as well as their noise spectra and statistics for joint optimization of their DSL parameters such as the lines' power-spectral densities (PSDs). This is in contrast to level-1 DSM where the SMC individually optimizes each line based on its own channel and noise conditions. The problem of jointly optimizing the lines' PSDs, referred to as *spectrum balancing*, has received much attention in recent literature, resulting in many novel algorithms such as in [3][4][5][6][7][8][9].

DSLs typically experience time-varying noises, which include impulse and quasi-stationary noises. The latter noise, which stays relatively stationary over a long time (on the orders of hours) but then changes to a different noise spectrum, is caused by varying crosstalk that occurs when other lines in the binder turn on/off or adjust their PSDs, and by noises from external devices such as TV, microwave, etc. The burst errors caused by impulse noise are best handled using forward-error-correction codes, while the SMC can optimize the margin parameters of the lines to improve their robustness against the quasi-stationary noise. However, the spectrum balancing problem is usually formulated assuming an ad-hoc signal-to-interference-and-noise-ratio (SINR) margin value for each line, and little work is available on joint margin and spectrum optimization in DSM. The authors in [10] considered maximizing the minimum SINR margin across lines by jointly optimizing their margins and PSDs, where the solution is achieved by allocating equal SINR-margins to all lines. However, this will result in varying degrees of robustness since each line typically has a different noise spectrum at initialization and a different degree of noise variation during online operation. Therefore, allocating equal SINR margins does not provide a robust operating point as even a few unstable lines may cause variable crosstalk to other lines, thus affecting the entire binder.

This paper investigates two aspects of the margin parameter in level-2 DSM: the definition of margin itself and the distributed implementation of joint margin and spectrum optimization. In level-2 DSM, since the PSDs of the lines are known to the SMC and are not expected to arbitrarily increase after initialization, the margins are primarily required for protection against possible increases in the crosstalk caused by other lines not managed by the SMC and other alien noise. This observation motivates a new definition of *noise margin* in Section III. The concept of probability of outage is introduced in Section IV to quantify a line's stability and throughput-guarantee using its empirical noise-variance statistics, initialization noise-spectrum, and noise margin. Section V presents an algorithm to jointly optimize the noise margins and PSDs to minimize the maximum outage probability among lines. Since the margin influences a line's power and hence the crosstalk it causes, joint margin and spectrum optimization provides improved politeness compared to spectrum optimization using ad-hoc SINR margins. The improved politeness provides as much as 100% improvements in data-rates or equivalently reduces the outage probability by an order of magnitude, thus improving the robustness of the system.

II. System Model

Figure 1 shows a DSL system where a central office (CO) or a remote terminal (RT) serves users using copper-lines that share the same binder. Let the total number of lines in the binder be K. A subset of U lines, $\mathcal{U} \subseteq \{1, 2, \ldots, K\}$, called the *managed lines*, is monitored by an SMC, which jointly optimizes the lines' DSL parameters. The remaining $K - U$ lines, which may be served by other service providers in an unbundled environment, are called the *unmanaged lines*.

The managed DSL lines employ discrete multi-tone transmission where an inter-symbol-interference (ISI) channel has

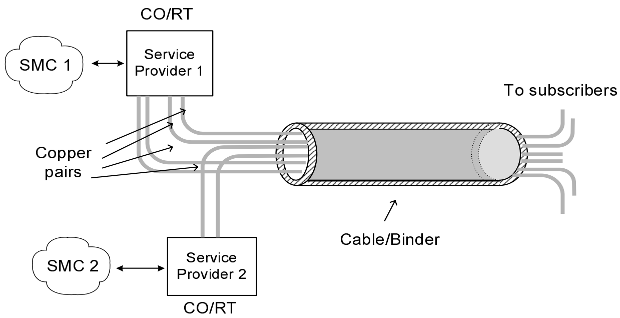

Fig. 1. A typical DSL system where the lines are monitored by SMCs.

been converted into N orthogonal, nearly-flat tones in the frequency domain. Line u's direct channel on tone n is denoted by $H^n_{u,u}$, while the far-end-crosstalk channel from line v to line u is denoted by $H^n_{u,v}$. The variance of the *alien-noise* experienced by line u on tone n, which is defined as the noise excluding the crosstalk from the other managed lines, is denoted by $(\sigma^n_u)^2$. The per-tone noise variance in dBm is modeled by a random process, $X^n_u(t)$, whose marginal distribution is empirically obtained by the SMC using long-term observations of each line. If the crosstalk from the managed lines is also included in the noise term, then the respective notations are $(\tilde{\sigma}^n_u)^2$ and $\tilde{X}^n_u(t)$.

III. Margin in Level-2 DSM Systems

The role of margin is to provide robustness against unexpected changes in the channels and noise-spectra. The margin value determines how much noise-increase can be withstood until the probability of error increases above the allowed limit.

A. SINR margin

Margin is usually applied as a reduction factor in the SINR. When treating all crosstalk as noise, the SINR of line u on tone n is $\mathrm{SINR}^n_u = \frac{\mathcal{E}^n_u |H^n_{u,u}|^2}{(\tilde{\sigma}^n_u)^2}$, and the number of bits is

$$b^n_u = \log_2\left(1 + \frac{\mathrm{SINR}^n_u}{\tilde{\gamma}_u \Gamma}\right), \quad (1)$$

where $\tilde{\gamma}_u$ is the *SINR margin* and Γ is the SNR gap of the code [11]. Equation (1) provides the number of bits on tone n that can be sustained at the desired probability-of-error (specified by Γ) if the SINR decreases by at most $\tilde{\gamma}_u$. A higher SINR margin provides protection against larger noise increases.

B. Noise margin

In level-2 DSM, the SINR can be explicitly written as

$$\mathrm{SINR}^n_u = \frac{\mathcal{E}^n_u |H^n_{u,u}|^2}{(\tilde{\sigma}^n_u)^2} = \frac{\mathcal{E}^n_u |H^n_{u,u}|^2}{\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}^n_v |H^n_{u,v}|^2 + (\sigma^n_u)^2}, \quad (2)$$

where the PSDs, $\mathcal{E}^n_u$, of the managed lines are fixed via coordination by the SMC. The fixed-PSD assumption is reasonable since DSLs usually maintain their PSDs after initialization. This paper assumes that if the noise-spectra change after initialization, the managed lines maintain their PSDs and simply adapt their data-rates using a mechanism such as the seamless-rate-adaptation (SRA) procedure in VDSL. However, the margin optimization framework presented in this paper is broad and can also cover situations when SRA is not used.

A multiplicative factor, γ^{no}_u, can be applied to the alien-noise variance, $(\sigma^n_u)^2$, to provide robustness against increases in the alien noise-variance. In addition, changes in the temperature or soil-conditions can cause variations in the channels. However, one can expect all channel transfer functions in a binder to be affected by roughly the same factor because the cross-section of a binder is very small compared to area affected by the large-scale phenomena. Assume that the direct and crosstalk channels decrease to $\frac{|H^n_{u,v}|^2}{\gamma^{\mathrm{ch}}_u}$. When incorporated into (2), both the robustness-factors appear as an effective multiplicative factor, γ_u, for the alien-noise variance.

$$b^n_u = \log_2\left(\frac{\mathcal{E}^n_u |H^n_{u,u}|^2}{\Gamma\left(\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}^n_v |H^n_{u,v}|^2 + \gamma_u (\sigma^n_u)^2\right)}\right), \quad (3)$$

where $\gamma_u = \gamma^{\mathrm{ch}}_u \gamma^{\mathrm{no}}_u$ is the *noise margin*, which is defined as the maximum tolerable increase in the alien-noise variance while maintaining the probability-of-error below the allowed limit[1]. Employing noise margins provides a formal method to reduce margin values especially when the managed crosstalk, which is assumed to be fixed or to have a maximum limit known to the SMC, is significant compared to the alien-noise.

1) Conversion between noise and SINR margins: Since DSL channels are usually static, noise-spectrum variations are the focus of this paper. Applying the noise margin requires separate knowledge of the managed crosstalk, $\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}^n_v |H^n_{u,v}|^2$, and the alien noise, $(\sigma^n_u)^2$. In level-2 DSM, such separation is possible at the SMC, which has knowledge of the crosstalk channels as well as the managed-users' PSDs. However, the noise margins should be converted into SINR margins for use by the modems, which cannot distinguish between managed crosstalk and the alien noise.

For fixed channels, alien-noise spectra, and PSDs, $\mathcal{E}^n_u$, the noise margin can be related to a per-tone SINR margin, $\tilde{\gamma}^n_u$, by equating (3) with $b^n_u = \log_2\left(\frac{\mathcal{E}^n_u |H^n_{u,u}|^2}{\Gamma \tilde{\gamma}^n_u (\tilde{\sigma}^n_u)^2}\right)$.

$$\tilde{\gamma}^n_u = \frac{\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}^n_v |H^n_{u,v}|^2 + \gamma_u (\sigma^n_u)^2}{\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}^n_v |H^n_{u,v}|^2 + (\sigma^n_u)^2}. \quad (4)$$

The SMC needs to provide the modem with an SINR margin that is the equal for all tones to be compatible with existing standards and to avoid the overhead required to communicate per-tone margins to the modems. The *equivalent SINR margin* that is equal across tones can be obtained by executing a bisection algorithm as in [12] using the per-tone SINR margins from (4). Similarly, a given SINR margin, $\tilde{\gamma}_u$, can be converted to a per-tone noise margin, γ^n_u by inverting (4), and replacing $\tilde{\gamma}^n_u$ with $\tilde{\gamma}_u$ and γ_u with γ^n_u. A formal definition of the *equivalent noise margin*, which can also be obtained by bisection similar to the equivalent SINR margin in [12], is

[1] Traditionally, margin has been interchangeably referred to as *SINR margin* or *noise margin*. In this paper, the two terms have different definitions.

Definition 1: For fixed $\mathcal{E}_u^n$, $H_{u,v}^n$, and γ_u^n $\forall u, v, n$, let $b_u^n = \log_2\left(1 + \frac{\mathcal{E}_u^n |H_{u,u}^n|^2}{\Gamma(\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}_v^n |H_{u,v}^n|^2 + \gamma_u^n (\sigma_u^n)^2)}\right)$ and $b'^n_u = \log_2\left(1 + \frac{\mathcal{E}_u^n |H_{u,u}^n|^2}{\Gamma(\sum_{v\in\mathcal{U}, v\neq u} \mathcal{E}_v^n |H_{u,v}^n|^2 + \gamma_u (\sigma_u^n)^2)}\right)$. The *equivalent noise margin* is the maximum γ_u such that $\sum_n b'^n_u = \sum_n b_u^n$. In other words, with fixed channels, noise spectra, and energies, the *equivalent noise margin* is the noise-margin, γ_u, that can be applied equally to all tones, while maintaining the data rate obtained using the unequal per-tone noise-margins, γ_u^n.

A closed-form approximation for the equivalent SINR or noise margin is obtained by averaging the per-tone SINR or noise margins in dB. The approximation for the equivalent SINR margin is very accurate [12], while simulations showed a 1 to 2 dB error range in the approximation for the equivalent noise margin. Figure 2 summarizes the conversions between the different per-tone and equivalent margin terms.

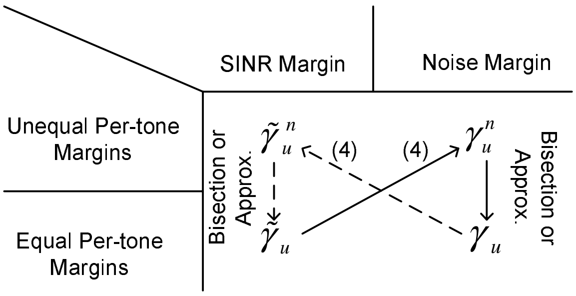

Fig. 2. Conversions between the various margin terms

2) Distributed enforcement of noise margin: The goal in level-2 DSM is for the SMC to help modems attain the desired operating point in a distributed manner to reduce complexity and overhead, and to allow quick adaptation to noise variations. Towards this goal, the SMC first executes the loading algorithms using the desired noise margin, γ_u^{des}, by multiplying it with the alien-noise spectrum, and obtains the PSDs, $\mathcal{E}_u^n$ of the managed lines. Then, the SMC calculates the per-tone SINR margins, $\tilde{\gamma}_u^n$, using (4), and determines the equivalent SINR margin, $\tilde{\gamma}_u$, using bisection. These equivalent SINR margins are then sent to the modems, which execute the loading algorithms and enter online operation.

However, the loading at the modems using the equivalent SINR margins can result in slightly different PSDs, $\hat{\mathcal{E}}_u^n$, than those determined by the SMC, $\mathcal{E}_u^n$, using the noise margins, γ_u^{des}. This can happen because the managed crosstalk obtained by executing the loading algorithm using the SINR margins, $\tilde{\gamma}_u$, can be larger than that obtained by the SMC using the noise margins, γ_u^{des}. This difference may cause the equivalent noise margin, $\hat{\gamma}_u$, to be lower than the desired margin for some users, which will consequently fail to maintain the desired robustness. To remedy this mismatch, the SMC can follow the additional steps in Algorithm 1. The SMC iteratively calculates the equivalent SINR margin using the new PSDs $\hat{\mathcal{E}}_u^n$ and the desired noise margin, γ_u^{des}, and performs loading in the same way as the modems will, using the equivalent SINR margins, until all the equivalent noise margins meet the desired values. The procedure may result in slightly lower data rates, $\hat{R}_u$, compared to the initial estimate, R_u. However, the loss in data rate was found to be negligible in the simulations.

Algorithm 1 Procedure followed by SMC to determine the noise margins to enable their distributed enforcement

1: Set γ_u^{des} equal to the desired values of noise margins for the managed lines $u \in \mathcal{U}$.
2: Execute the loading algorithms using noise margin, γ_u^{des}. Obtain the PSDs, $\mathcal{E}_u^n$ $\forall u, n$, and the data rates, R_u, $\forall u$.
3: **loop**
4: Obtain the equivalent SINR margins, $\tilde{\gamma}_u$, from $\mathcal{E}_u^n$ $\forall u, n$ and γ_u^{des} using (4) and bisection.
5: Execute the loading algorithms using the SINR margins, $\tilde{\gamma}_u$. Obtain the resulting PSDs, $\hat{\mathcal{E}}_u^n$ $\forall u, n$, and the data rates, $\hat{R}_u$, $\forall u$.
6: Obtain the equivalent noise margins, $\hat{\gamma}_u$ by bisection.
7: Set $\mathcal{E}_u^n = \hat{\mathcal{E}}_u^n$ $\forall u, n$.
8: **if** $\hat{\gamma}_u < \gamma_u^{\text{init}}$ for some u **then**
9: Return to Step 4.
10: **else**
11: Exit the loop.

IV. Probability of Outage

A stable data-rate that can be guaranteed with a high probability is important for providing QoS in a broadband communication system. This requirement is captured by:

Definition 2: The *probability of outage*, $P_{out,u}$, is the probability that the line u can only operate below the target data rate when forced to meet the target probability of error.

The probability of outage is very useful in determining the QoS of a line. For example, a service provider may require that a line should be able to operate at 30 Mbps for 99% of the time with a bit-error-rate of 10^{-7} to provide a HDTV service. This can be translated into a requirement that $P_{out,u} < 0.01$.

Since the PSDs of the managed lines are assumed to be fixed after initialization, a managed line will experience outage if the alien-noise spectrum increases by more than the allocated initialization noise-margin, γ_u^{init}. An outage event can equivalently be viewed as the noise margin becoming negative after initialization. Therefore, the probability of outage can be defined as $P_{out,u} = Pr\{\Upsilon_u(\text{dB}) < 0\}$, where the random variable Υ_u represents line u's noise margin variation after initialization. The goal is to determine $P_{out,u}$ as a function of γ_u^{init} for use in the performance optimization. First, the per-tone noise-margins are considered as random variables, Υ_u^n. Maintaining the PSD after initialization and before bit-swapping, the variation in per-tone noise margins caused by variations in the alien-noise spectrum is given by

$$\Upsilon_u^n(\text{dB}) = \gamma_u^{\text{init}}(\text{dB}) + (\sigma_u^{n,\text{init}})^2(\text{dBm}) - X_u^n(\text{dBm}), \quad (5)$$

where $(\sigma_u^{n,\text{init}})^2$ is the per-tone alien-noise-variance at initialization of line u, and X_u^n is the random variable representing the current alien-noise-variance on tone n for line u.

To calculate the probability of outage, the equivalent noise margin, Υ_u, is obtained using the approximation described before, i.e., $\Upsilon_u(\text{dB}) = \frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} \Upsilon_u^n(\text{dB})$, where $\mathcal{N}_{\text{used}} \subseteq \{1,\ldots,N\}$, (of size N^*), is the set of tones to which energy allocated during bit-loading. The approximation works well when either the alien noise is much larger than the managed crosstalk or vice-versa. When the managed crosstalk is on the order of the alien noise, the approximation can differ from the actual value by a couple of dB. Using the approximation,

$$
\begin{aligned}
P_{out,u} &= Pr\Big\{\Upsilon_u(\text{dB}) < 0\Big\} = Pr\Big\{\frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} \Upsilon_u^n(\text{dB}) < 0\Big\} \\
&= Pr\Big\{\frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} X_u^n(\text{dBm}) > \gamma_u^{\text{init}}(\text{dB}) + \\
&\qquad \frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} (\sigma_u^{n,\text{init}})^2(\text{dBm})\Big\}. \qquad (6)
\end{aligned}
$$

The metric captures the intuition that for a desired $P_{out,u}$, the noise margin can be reduced if the alien-noise is larger than its nominal value. The SMC can obtain the statistics of $\hat{X}_u = \frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} X_u^n$ by fitting the long-term average alien-noise-variance observations to an appropriate distribution.

V. Optimization of Margins in Level-2 DSM

In level-2 DSM, the initialization-margins for the managed lines can be jointly optimized by the SMC using both the initialization noise-spectrum and the distribution of the noise-variance, instead of allocating ad-hoc margin values. Thus, margins are allocated considering the long-term stability and throughput-guarantee of the services of all the managed lines.

One approach for margin optimization in level-2 DSM is to assign target data-rates to the users and then minimize the maximum probability of outage across them. The min-max criterion is chosen since it reflects the physical reality in the system. While each user would like its probability of outage to be minimized, it is important to ensure that there are not many unstable lines in the group managed by the SMC. Lines having a high probability of outage will tend to re-initialize often, thus causing variations in the managed crosstalk. This in turn may cause instability issues in other managed lines, thus causing a cascading failure of the system. Therefore, minimizing the maximum probability of outage among the users in a level-2 setting has a system-wide stability interpretation. The problem of joint optimization of the users' margins and PSDs is:

$$
\begin{aligned}
&\text{minimize} \quad \max_{u\in\mathcal{U}} P_{out,u} \\
&\text{subject to} \quad \sum_{n=1}^{N} \mathcal{E}_u^n \le \mathcal{E}_u^{\text{tot}},\ \forall u\in\mathcal{U} \\
&\qquad \mathcal{E}_u^n \ge 0,\ \forall u\in\mathcal{U},\ \forall n \\
&\qquad \gamma_u^{\text{init}} \ge 1,\ \forall u\in\mathcal{U} \\
&\sum_{n=1}^{N}\log_2\left(1+\frac{\mathcal{E}_u^n|H_{u,u}^n|^2}{\Gamma\left(\sum_{v\in\mathcal{U},v\neq u}\mathcal{E}_v^n|H_{u,v}^n|^2+\gamma_u^{\text{init}}(\sigma_u^{n,\text{init}})^2\right)}\right) \\
&\qquad = R_u^{\text{target}},\ \forall u\in\mathcal{U}, \qquad (7)
\end{aligned}
$$

where the optimization variables are the noise margins, γ_u^{init}, and the energy allocations, $\mathcal{E}_u^n$, which are selected while satisfying constraints on the target-data-rate and the sum-energy across tones for each user, and a requirement that the noise margins should be at least 0 dB (1 in linear scale).

The objective in (7) can be converted into U constraints, $P_{out,u} \le y,\ \forall u \in \mathcal{U}$, with the new objective being to minimize y. Let $F_{\hat{X}_u}^{-1}(\cdot)$ be the inverse cumulative distribution function (cdf) of $\hat{X}_u$. Using (6), these outage probability constraints, $P_{out,u} \le y,\ \forall u \in \mathcal{U}$, along with the minimum 0 dB margin constraints, can be replaced by $\gamma_u^{\text{init}} \ge g_u(y),\ \forall u \in \mathcal{U}$, where $g_u(y) = \max\Big(1, F_{\hat{X}_u}^{-1}(1-y) - \frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}}(\sigma_u^{n,\text{init}})^2(\text{dBm})\Big)$. Therefore, the equivalent problem is

$$
\begin{aligned}
&\text{minimize} \quad y \\
&\text{subject to } \gamma_u^{\text{init}} \ge g_u(y),\ \forall u\in\mathcal{U} \\
&\sum_{n=1}^{N} \mathcal{E}_u^n \le \mathcal{E}_u^{\text{tot}},\ \forall u\in\mathcal{U},\ \mathcal{E}_u^n \ge 0,\ \forall u\in\mathcal{U},\ \forall n \\
&\sum_{n=1}^{N}\log_2\left(1+\frac{\mathcal{E}_u^n|H_{u,u}^n|^2}{\Gamma\left(\sum_{v\in\mathcal{U},v\neq u}\mathcal{E}_v^n|H_{u,v}^n|^2+\gamma_u^{\text{init}}(\sigma_u^{n,\text{init}})^2\right)}\right) \\
&\qquad = R_u^{\text{target}},\ \forall u\in\mathcal{U}, \qquad (8)
\end{aligned}
$$

Since $g_u(\cdot)$ is a decreasing function because the inverse cdf function is an increasing function, y is minimized when $\gamma_u^{\text{init}} = g_u(y),\ \forall u \in \mathcal{U}$, assuming $g_u(y)$ is continuous. Therefore, the problem in (8) can be solved by the bisection procedure in Algorithm 2. For a given y, the margins, $\gamma_u^{\text{init}},\ \forall u \in \mathcal{U}$, are determined using $g_u(\cdot)$. The remaining problem is to check the feasibility of the data-rates, R_u^{target}, using these margins:

$$
\begin{aligned}
&\text{minimize} \quad 0 \\
&\text{subject to} \quad \sum_{n=1}^{N} \mathcal{E}_u^n \le \mathcal{E}_u^{\text{tot}},\ \forall u\in\mathcal{U} \\
&\qquad \mathcal{E}_u^n \ge 0,\ \forall u\in\mathcal{U},\ \forall n, \\
&\sum_{n=1}^{N}\log_2\left(1+\frac{\mathcal{E}_u^n|H_{u,u}^n|^2}{\Gamma\left(\sum_{v\in\mathcal{U},v\neq u}\mathcal{E}_v^n|H_{u,v}^n|^2+g_u(y)(\sigma_u^{n,\text{init}})^2\right)}\right) \\
&\qquad = R_u^{\text{target}},\ \forall u\in\mathcal{U}. \qquad (9)
\end{aligned}
$$

The feasibility problem described by (9) can be solved by fixing the data-rate targets for any $U-1$ of the U users and maximizing the data rate of the U^{th} user using any existing spectrum balancing algorithm. If the resulting data rate for the U^{th} user is greater than or equal to its target, then the problem is feasible. Otherwise, the problem is infeasible.

An alternate formulation for joint margin and spectrum optimization in level-2 DSM is to maximize the weighted sum-rate of the users subject to outage probability constraints. The solution to this problem in [13] also leverages the noise margin concept and existing spectrum balancing algorithms.

Distributed implementation of the optimized noise margins and PSDs is achieved by following Algorithm 1. Steps 1 and 2

Algorithm 2 Bisection method to solve for min-max Pout problem

Initialize $y_{\min} = 0$ and $y_{\max} = 1$.
Choose a tolerance $\epsilon_y > 0$
repeat
  $y_{\text{avg}} = (y_{\min} + y_{\max})/2$
  if Problem (9) is feasible with $y = y_{\text{avg}}$ **then**
    $y_{\max} = y_{\text{avg}}$
  else
    $y_{\min} = y_{\text{avg}}$
until $|y_{\max} - y_{\text{avg}}| < \epsilon_y$
$y = y_{\text{avg}},\ \gamma_u^{\text{init}} = g_u(y),\ \forall u \in \mathcal{U}$

should be replaced by solving the optimization problem in (7) and setting γ_u^{des} to the optimal value of γ_u^{init}. The remaining steps remain the same. The optimal value of the equivalent initialization SINR margins are then sent to the modems, which can then execute their loading algorithms using the near-distributed spectrum balancing algorithms such as Band-preference[7].

Finally, practical DSL systems may require that the PSDs of the managed lines have some flexibility to vary during online operation. To allow such variations, an extra margin parameter, with a small value, can be applied to the managed crosstalk in the data-rate equations. If the SMC has knowledge of the amount of PSD variation during online operation, the extra margin parameter can also be optimized.

VI. Numerical Results

This section presents numerical results to illustrate the benefits of the noise margin concept and the proposed margin optimization algorithms. A 26-guage, VDSL2 service is considered. A 3 dB coding gain and 9.8 dB uncoded gap were used. A model using Beta distributions, developed in [12], was used for the alien-noise-variance distribution for each tone, which also incorporates the alien-noise-variance correlation across tones. The minimum noise spectrum was chosen to be a constant -140 dBm/Hz AWGN and the maximum noise spectrum was chosen to be an ANSI Noise A PSD [14], which is a mixture of 16 ISDN, 4 HDSL, and 10 ADSL disturbers, with a modified noise floor of -119 dBm/Hz. The shape parameters of the Beta distributions were chosen to be $a_u^n = b_u^n = 4$ and the correlation coefficient of the noise-variance between adjacent tones was chosen to be 0.9. This value is reasonable since the noise powers of adjacent tones are expected to be highly correlated. Moreover, the chosen noise model has a property that the correlation across tones reduces exponentially as the tone-separation increases.

First, the iterative-water-filling (IWF) algorithm [3] was executed for 10 downstream users with equal loop lengths. The initialization alien-noise-spectra for the users were randomly and independently chosen between the minimum and maximum levels in the Beta model. Figure 3 shows around $40-50\%$ improvement for short loops in the average data-rate among users by using a 6 dB noise margin instead of a 6 dB SINR margin. As the looplength increases, the crosstalk becomes less dominant and finally the use of noise margin and SINR margin becomes identical at long loop lengths since the alien-noise dominates the managed crosstalk. The corresponding effective SINR margins are shown in Fig. 4. As intuitively expected, the effective SINR margins are very small when the managed crosstalk dominates in short loops. For longer loops, where the alien-noise dominates and its variations become important, the SINR margin increases back to the 6 dB value.

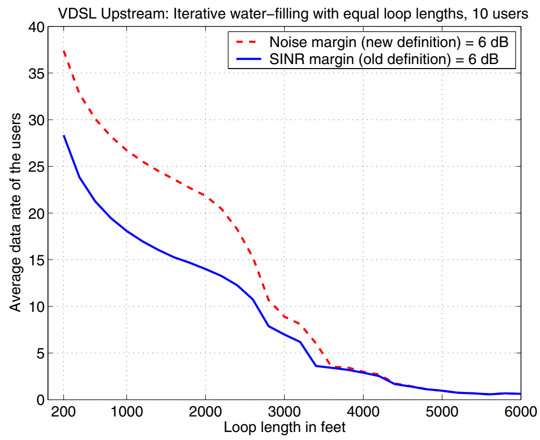

Fig. 3. Increase in average data-rate in IWF by using 6 dB noise margins for all lines compared to 6 dB SINR margins

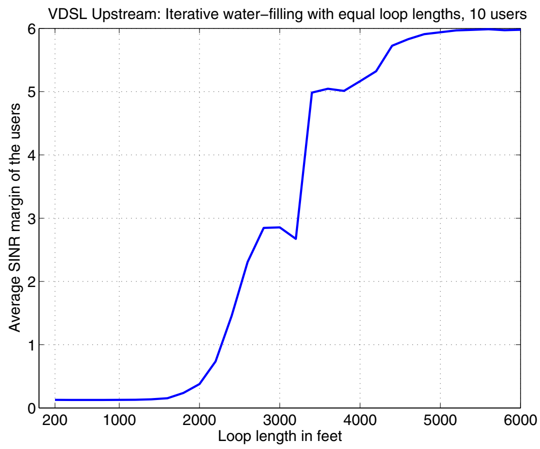

Fig. 4. Effective SINR margins corresponding to the 6 dB noise-margin curve in Figure 3

Figure 5 shows the improvement in the optimal rate-region, provided by the optimal spectrum balancing (OSB) algorithm [4], using a 6 dB noise margin instead of a 6 dB SINR margin for two equal-length VDSL2 lines. The initialization alien-noise-spectra for the users was chosen randomly between the minimum and maximum levels, with user 1 having a larger alien-noise spectrum. The use of noise margin provides gains of about 50% in the users' data-rates, especially in the region where both lines are operating at high data-rates.

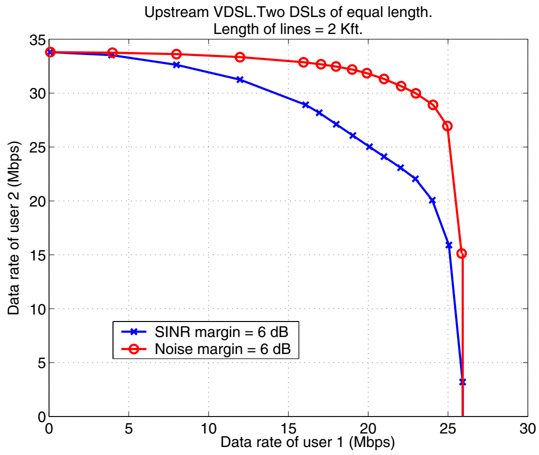

Fig. 5. Improvement in data-rate region by using 6 dB noise margins instead of using 6 dB SINR margins

Next, the improvements with optimized margins are considered. When the data-rate pairs from the 6 dB SINR margin OSB region from Fig. 5 are chosen and the outage probability problem in (7) is solved, Fig. 6 is obtained. The maximum outage probability across users decreases by orders of magnitude compared to the ad-hoc 6 dB SINR margin case. The improvement in outage probabilities is observed for the rate-tuple indices 5 through 15 where the interference between users 1 and 2 is significant. Thus, optimization of the margins across users can be viewed as a rate-region enhancement or an improvement in the users' quality-of-service (in terms of outage probability) via improved politeness in the binder.

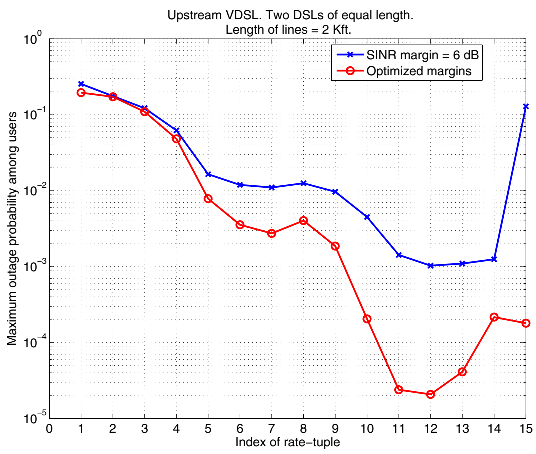

Fig. 6. Reduction in outage probabilities by optimizing margins corresponding to the 15 rate-pairs in the 6 dB SINR-margin-rate-region in Figure 5
.

For rate-tuples where the interference between the two users is low, for example rate-tuple indices 1 to 4, Fig. 6 shows high outage probabilities. The reason for these high outage probabilities is that the choice of 6 dB margin was a poor operating point to begin with. A lower outage probability can be obtained in this region by solving a weighted-sum-rate problem subject to outage probability constraints.

VII. Conclusions

A holistic framework for joint margin and spectrum optimization in level-2 DSM systems has been presented. The new definition of noise margin and the concept of probability of outage enabled an easier formulation of the problems of jointly optimizing the lines' PSDs and margins, while also considering their stability. Using these concepts, an algorithm was presented to jointly optimize the margins and PSDs to provide maximum robustness to the lines against quasi-stationary noise variations. The presented solution effectively decouples the joint margin and spectrum optimization problem to leverage the existing spectrum balancing algorithms along with the use of the new margin optimization procedures. In this manner, the effects of interference and quasi-stationary noise can be significantly mitigated in level-2 DSM. One can also envision extending the principles developed to other multi-user communication systems that need to deal with interference and also employ margins to protect against unexpected changes in the channel conditions.

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