PER-TONE MARGIN OPTIMIZATION IN MULTI-CARRIER COMMUNICATION SYSTEMS

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ABSTRACT

This paper investigates the unequal allocation of margin across tones in a multicarrier communication system. Using empirical statistics of the channel signal-to-noise-ratio variation, an algorithm is developed for modems to optimally distribute the available margin across tones instead of allocating equal per-tone margins. Such unequal per-tone margin allocation results in better stability of the line in terms of the outage probability especially when bit and energy adaptation mechanisms are not available or respond slowly to changes in the channel or noise spectrum.

***Index Terms*—** Multi-carrier, Margin, Stability, Outage

1. INTRODUCTION

Multicarrier modulation is being extensively employed in various wireline and wireless systems to communicate efficiently over frequency selective channels. In addition, multicarrier modulation is used in the presence of temporal variation of the signal-to-noise ratio (SNR). In digital-subscriber-lines (DSLs), the channels can be assumed to be static while the noise can be modeled to be quasi-stationary, i.e., the noise stays relatively stationary over a long time (on the order of hours), but then changes to a different noise spectrum. Such noise is caused by other users in the telephone-line binder turning on/off or adjusting their transmit spectra, or by noises from external devices such as TV, microwave, etc. Similar noise variations may also be present in a power-line-communication (PLC) system. In addition, an in-home PLC channel also experiences quasi-static variations of the channel or noise when a bridged-tap impedance changes because of a device turning on/off. In a slowly-fading wireless system, the noise is usually stationary, while the channel varies slowly relative to the symbol period. In general, the quasi-stationary variations in the channel and/or noise can be aggregately modeled as channel SNR variations. It is important to be robust against these slow channel-SNR variations to provide a reliable service.

To cope with these SNR variations, multicarrier systems have the ability to adapt to slowly-changing channel and noise conditions by using, for example, the bit- and gain-swapping procedures in digital-subscriber-lines (DSLs) [1], or adaptive modulation and coding in wireless systems. In addition, to assist the adaptation, a signal-to-noise ratio (SNR) margin parameter, which is the decrease in the SNR that can be withstood by the system at the target data-rate and error-probability, is used in multicarrier communication systems such as DSLs. The adaptation procedures and the margin parameter are especially useful for supporting emerging applications such as video, HDTV, VoIP, etc., which impose more strict quality-of-service (QoS) requirements such as low delay and high stability.

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Typically, the SNR margin parameter for each modem is allocated at initialization by a central system and is applied equally to all used tones. The optimization of these initialization margins, $\tilde{\gamma}_u^{\text{init}}$, is presented in [2], where only one margin value is shared by all tones to maintain compatibility with current DSL standards and to reduce the control overhead. However, allocating equal margins to all tones would result in unequal protection of different tones since the degree of channel-SNR variation may be different across tones. For example, DSL systems usually experience larger noise variations caused by time-varying crosstalk at higher frequencies or by radio-frequency interference (RFI) at certain frequencies.

This unequal noise variation after initialization can make the margin of the affected tones to be negative. If the minimum margin across all tones remains negative for a long time, then the error probability of the system will exceed its target. Therefore, DSL modems usually re-initialize after a timeout period (around 30 secs) if the minimum per-tone margin remains negative. In many situations, adaptation mechanisms such as bit-swapping[1] can restore the per-tone margins to non-negative values given sufficient time. However, some systems may not use bit-swapping, while others may use bit-swapping but the procedure might be too slow. In such cases, it is prudent to protect the weakest link by providing higher margins to tones that are more susceptible to a noise increase thereby providing short-term stability in the DSL link by reducing the occurrence of negative margin events. With this motivation, this paper develops an algorithm to re-distribute the available margin unequally across tones using knowledge of the channel SNR distribution, with the aim of minimizing the probability that the lowest margin across tones becomes negative.

2. SYSTEM MODEL

A system employing discrete multi-tone (DMT) transmission or orthogonal frequency division multiplexing (OFDM) is considered, where an inter-symbol-interference (ISI) channel has been converted into N orthogonal tones (or subchannels) in the frequency domain. On tone n for user u, the channel and the noise-variance, which includes the crosstalk from other users, are respectively denoted by $H_{u,u}^n$ and $(\tilde{\sigma}_u^n)^2$. User u's channel SNR on tone n is denoted by g_u^n, where $g_u^n = \frac{|H_{u,u}^n|^2}{(\tilde{\sigma}_u^n)^2}$. During online operation, the time-varying channel SNR, in dB, for user u on tone n is modeled by a random process $G_u^n(t)$, whose first-order distribution is obtained empirically using long-term observations of the channel SNR. For notational convenience, the time index is ignored in further sections. Such a distribution may be obtained by the user's modem itself or by an external system such as a spectrum management center [3].

[1]Bit-swapping moves bits across tones to equalize the margin across tones, and thus maximize the minimum margin across tones.

3. SNR MARGIN AND PROBABILITY OF OUTAGE

The role of margin in communication systems is succinctly explained by the following statement from [4]: "The margin specification is a way of stating that there must be an ability for the technology to withstand impairments that are not normally anticipated." Traditionally, margin has been used to determine how much decrease in the SNR can be withstood until the probability of error increases above the maximum value. If $\mathcal{E}_u^n$ is the energy allocated to user u on tone n, then $\text{SNR}_u^n = \frac{\mathcal{E}_u^n |H_{u,u}^n|^2}{(\tilde{\sigma}_u^n)^2} = \mathcal{E}_u^n g_u^n$, and the number of bits is

$$b_u^n = \log_2\left(1 + \frac{\text{SNR}_u^n}{\tilde{\gamma}_u \Gamma}\right), \quad (1)$$

where $\tilde{\gamma}_u$ is the *SNR margin* and Γ is the SNR gap of the code [5]. Equation (1) provides the number of bits on tone n that can be sustained at the desired maximum probability-of-error (implicitly specified by Γ) if the SNR decreases by at most $\tilde{\gamma}_u$. A higher SNR margin provides more protection against larger variations in the channel SNR. The following defines a metric that quantifies the robustness of a user as a function of its SNR margin.

Definition 1 *The probability of outage, $P_{out,u}$, is the probability that the user u cannot operate at the target data rate at or below the target probability of error.*

The probability of outage is very useful in determining the QoS of a user. For example, a service provider may require that a user be able to operate at 30 Mbps for 99% of the time with a probability-of-error of 10^{-7} to provide a high-quality video service. Such a requirement can be translated into a $P_{out,u} < 0.01$ requirement.

The target probability-of-error of a user is determined by the effective SNR gap, which is the sum in dB of the SNR gap, Γ, and the SNR margin, $\tilde{\gamma}_u$. If the SNR margin is less than 0 dB, then the effective gap of the system is smaller than the gap required for the target error-rate. Hence, the target probability of error is no longer achievable at the target data rate, which means the user is in *outage*. With this observation, the definition of the probability of outage translates to the probability that the SNR margin drops below 0 dB, i.e., $P_{out,u} = Pr\left\{\tilde{\Upsilon}_u(\text{dB}) < 0\right\}$, where $\tilde{\Upsilon}_u$ is the random variable representing user u's SNR margin. This definition of the probability-of-outage is similar to the usual definition in wireless systems [6], except that an SNR margin is introduced to tune the system to achieve the desired outage probability.

This system-wide outage probability can be generalized to the per-tone outage probability to quantify the robustness of each tone. Let $\tilde{\gamma}_u^{n,\text{init}}$ be the SNR margin that is allocated to tone n of user u during initialization. The probability of outage on tone n, $P_{out,u}^n$, is determined by considering the per-tone margins during online operation as random variables, $\tilde{\Upsilon}_u^n$. Maintaining the same number of bits and energy on each tone after initialization, variations in the channel SNR during online operation will cause the margins to vary, which gives:

$$\tilde{\Upsilon}_u^n(\text{dB}) = \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) - g_u^{n,\text{init}}(\text{dBm}) + G_u^n(\text{dBm}), \quad (2)$$

where on tone n for user u, $g_u^{n,\text{init}}$ is channel SNR at initialization and G_u^n is the random variable representing the current channel SNR. Therefore, the probability-of-outage on tone n, $P_{out,u}^n$, is

$$\begin{aligned} P_{out,u}^n &= Pr\left\{\tilde{\Upsilon}_u^n(\text{dB}) < 0\right\} \\ &= Pr\left\{G_u^n(\text{dBm}) < g_u^{n,\text{init}}(\text{dBm}) - \tilde{\gamma}_u^{n,\text{init}}(\text{dB})\right\} \\ &= F_{G_u^n}\left(g_u^{n,\text{init}}(\text{dBm}) - \tilde{\gamma}_u^{n,\text{init}}(\text{dB})\right), \end{aligned} \quad (3)$$

where $F_{G_u^n}(\cdot)$ is the cumulative distribution function (cdf) of the channel SNR, G_u^n, in dB. Using $F_{G_u^n}(\cdot)$, the per-tone margins can be optimized by each modem at initialization, to provide equal robustness, i.e. outage probabilities, to all tones when the channel SNR changes during online operation.

4. OPTIMIZATION OF THE PER-TONE SNR MARGINS

This section develops an algorithm for the modem to optimally re-distribute the available SNR margin unequally across tones to provide equal stability, in terms of the outage probability, for each tone. Tone-dependent margins were first proposed in [7], applying to any or all margin parameters. Virtual noise (VN) [8] is an example of tone-dependent margin where the per-tone margin is enlarged by the difference between a nominal noise and the VN, which may be close to the worst-case noise level. For various reasons such as increased transmit power and hence, increased crosstalk to other users, the VN-type of frequency-dependent margins is detrimental to other users and causes network instability as various users fight with one another to maintain guard against potential power increases of the others.

However, another use of [7] is a per-tone target margin as in this paper, which can be shown to be more effective than VN approach. Care must be taken when providing unequal per-tone margins since increasing the margin on a tone by increasing the user's energy could cause more crosstalk to other users thereby bringing down their data rates or making other users unstable. In fact, since multiple users could share the same set of tones in the communication system, the power-spectral-densities (PSDs) of the users may have been optimized during initialization to operate at the desired data-rates, by smartly managing the crosstalk between the users in the system. For example, this can be achieved in DSL via *spectrum balancing* [1]. Hence, this paper assumes that the PSD obtained during initialization is fixed during online operation so that the crosstalk originating from the user remains fixed.

In a DSL system, a central node provides the SNR margin $\tilde{\gamma}_u^{\text{init}}$ that should be used by the modem to achieve the desired QoS. Then, the modem can re-distribute the margin unequally across tones to minimize the maximum per-tone outage probability, while maintaining the initial PSD of the user. This re-distribution requires a relation between the equal and unequal per-tone SNR margins for maintaining the same data rate and energy allocation.

Definition 2 *Let $b_u^n = \log_2\left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u^n \Gamma}\right)$ for fixed $\mathcal{E}_u^n$, g_u^n, and $\tilde{\gamma}_u^n$ $\forall u, n$. The equivalent SNR margin is defined to be the maximum value of $\tilde{\gamma}_u$ such that ${b'}_u^n = \log_2\left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u \Gamma}\right)$ and $\sum_n {b'}_u^n = \sum_n b_u^n = R_u$. In other words, for a given channel, noise spectrum, and PSD, the equivalent SNR margin is the SNR margin, $\tilde{\gamma}_u$, that can be applied equally to all tones, while maintaining the data rate obtained using the unequal per-tone SNR margins, $\tilde{\gamma}_u^n$.*

Consider $f(\tilde{\gamma}_u) = \sum_n \log_2\left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u \Gamma}\right) - R_u$. Since $f(\tilde{\gamma}_u)$ is a decreasing and continuous function of $\tilde{\gamma}_u$, bisection can be used to solve for the value of $\tilde{\gamma}_u$ that results in $f(\tilde{\gamma}_u) = 0$. In addition, an approximate closed-form expression for the equivalent SNR margin can be derived. The condition $\sum_n {b'}_u^n = \sum_n b_u^n = R_u$ implies

$$\prod_n \left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u^n}\right) = \prod_n \left(1 + \frac{\mathcal{E}_u^n g_u^n}{\tilde{\gamma}_u}\right). \quad (4)$$

Neglecting the addition of 1 on both sides of (4), we obtain $\tilde{\gamma}_u \approx \left(\prod_n \tilde{\gamma}_u^n\right)^{\frac{1}{N}}$ in linear scale, or in the log scale

$$\tilde{\gamma}_u(\text{dB}) \approx \frac{1}{N}\sum_n \tilde{\gamma}_u^n(\text{dB}). \tag{5}$$

The approximation is accurate when the SNR is high, which is typical in DSL systems. In fact, exhaustive simulations indicated that even for low SNRs (0 to 10 dB), the approximation is within a 0.5 dB error range when the maximum deviation in the per-tone margins, $\tilde{\gamma}_u^n$, is less than 40 dB. This range is sufficient for most practical systems. An implicit assumption in the derivation of the approximation is that the energy on each tone, $\mathcal{E}_u^n$, is non-zero. In practice, only the set of tones, $\mathcal{N}_{\text{used}} \subseteq \{1,\ldots,N\}$, (of size N^*) to which energy is allocated during bit-loading should be considered.

The problem of minimizing the maximum outage-probability across tones can then be formulated as:

$$\begin{array}{ll} \text{minimize} & \max\limits_{n\in\mathcal{N}_{\text{used}}} P_{out,u}^n \\ \text{subject to} & \dfrac{1}{N^*}\sum\limits_{n\in\mathcal{N}_{\text{used}}} \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) = \tilde{\gamma}_u^{\text{init}}(\text{dB}) \\ & \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) \geq 0, \forall n \in \mathcal{N}_{\text{used}} \end{array}, \tag{6}$$

where the set $\mathcal{N}_{\text{used}} \subseteq \{1,\ldots,N\}$ contains the used-tone indices. This set is simply determined by running the bit-loading algorithm using the constant SNR margin, $\tilde{\gamma}_u^{\text{init}}$, which also determines the data-rate and PSD of the user. The equality constraint captures the goal of re-distributing the equivalent SNR margin, $\tilde{\gamma}_u^{\text{init}}$, into unequal per-tone margins, $\tilde{\gamma}_u^{n,\text{init}}$, which are the optimization variables. Since the spectrum and data rate of the user are kept unchanged after initialization even with the unequal re-distribution of the margin across tones, the equality constraint is obtained using the approximation in (5).

The optimization problem in (6) is equivalent to

$$\begin{array}{ll} \text{minimize} & y \\ \text{subject to} & P_{out,u}^n \leq y, \forall n \in \mathcal{N}_{\text{used}} \\ & \dfrac{1}{N^*}\sum\limits_{n\in\mathcal{N}_{\text{used}}} \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) = \tilde{\gamma}_u^{\text{init}}(\text{dB}) \\ & \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) \geq 0, \forall n \in \mathcal{N}_{\text{used}} \end{array}. \tag{7}$$

Using (3), the constraints $P_{out,u}^n \leq y$ are replaced by $\tilde{\gamma}_u^{n,\text{init}} \geq h_n(y)$ for each n, where $h_n(y) = g_u^{n,\text{init}}(\text{dBm}) - F_{G_u^n}^{-1}(y)$ is monotonically decreasing since the cdf $F_{G_u^n}(\cdot)$ is monotonically increasing. Consequently, $q_n(y) = \max\{0, h_n(y)\}$ is also monotonically decreasing in y, which implies that it is quasilinear. The equivalent optimization problem is then

$$\begin{array}{ll} \text{minimize} & y \\ \text{subject to} & q_n(y) - \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) \leq 0, \forall n \in \mathcal{N}_{\text{used}} \\ & \dfrac{1}{N^*}\sum\limits_{n\in\mathcal{N}_{\text{used}}} \tilde{\gamma}_u^{n,\text{init}}(\text{dB}) = \tilde{\gamma}_u^{\text{init}}(\text{dB}) \end{array}. \tag{8}$$

The constraint $q_n(y) \leq \tilde{\gamma}_u^{n,\text{init}}$ corresponds to a sub-level set of the quasilinear function $q_n(y)$. Since the sub-level sets of a quasilinear function are convex, the constraint can be replaced by equivalent convex constraint functions [9]. Therefore, (8) is a convex optimization problem. If the equivalent convex constraint functions can be determined by utilizing the structure of the cdfs $F_{G_u^n}(\cdot)$, then efficient convex optimization techniques can be employed. Since, such a structure may not always exist, a general solution to the problem, which still achieves the optimal objective value, is presented.

First note that (8) is always feasible since the margins, $\tilde{\gamma}_u^{n,\text{init}}$, can be set to be equal to $\tilde{\gamma}_u^{\text{init}}$ on all tones and y can be chosen to satisfy $q_n(y) \leq \tilde{\gamma}_u^{n,\text{init}}$. Let $q_n(y),\ \forall n \in \mathcal{N}_{\text{used}}$, be continuous functions of y, and let the optimal solution y^* be such that $q_n(y^*) < \tilde{\gamma}_u^{n,\text{init}}$. Then by the monotonicity and continuity of $q_n(y)$, y^* can be further decreased until the constraint is met with equality, which results in a contradiction. Hence, the optimal solution y^* should satisfy $q_n(y^*) = \tilde{\gamma}_u^{n,\text{init}}$. This equality is consistent with the intuition that the outage probabilities of all tones should be identical in order to minimize the maximum per-tone outage probability. Consequently, the two constraints in (8) can be combined to obtain

$$\begin{array}{ll} \text{minimize} & y \\ \text{subject to} & \dfrac{1}{N^*}\sum\limits_{n\in\mathcal{N}_{\text{used}}} q_n(y) = \tilde{\gamma}_u^{\text{init}}(\text{dB}) \end{array}. \tag{9}$$

The optimal solution to problem (9) can be obtained using bisection on the scalar y as described in Algorithm 1, since the functions $q_n(y)$ are continuous and monotonically decreasing.

Algorithm 1 Bisection method to distribute margin across tones

1: Initialize $y_{\text{min}} = 0$ and $y_{\text{max}} = 1$.
2: Choose a tolerance $\epsilon > 0$
3: **repeat**
4: $\quad y_{\text{avg}} = (y_{\text{min}} + y_{\text{max}})/2$
5: $\quad$ **if** $\frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} q_n(y_{\text{avg}}) \geq \tilde{\gamma}_u^{\text{init}}$ **then**
6: $\quad\quad y_{\text{min}} = y_{\text{avg}}$
7: $\quad$ **else**
8: $\quad\quad y_{\text{max}} = y_{\text{avg}}$
9: **until** $\left|\frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} q_n(y_{\text{avg}}) - \tilde{\gamma}_u^{\text{init}}\right| \leq \epsilon$
10: $y = y_{\text{avg}}$, $\tilde{\gamma}_u^{n,\text{init}} = q_n(y)$

If $q_n(y)$ is not a continuous function of y for some n, then Algorithm 1 can still be used with a few modifications. The stopping criterion in Step 9 should be replaced with $|y_{\text{max}} - y_{\text{avg}}| < \epsilon_y$, where the tolerance, $\epsilon_y > 0$, is chosen to be sufficiently small. Since $\frac{1}{N^*}\sum_{n\in\mathcal{N}_{\text{used}}} q_n(y_{\text{max}}) < \tilde{\gamma}_u^{\text{init}}$, Step 10 should be replaced by choosing $y = y_{\text{max}}$. Finally, there could be multiple choices for assigning the margin per tone. A simple choice would be to distribute the remaining margin equally across all tones $n \in \mathcal{N}_{\text{used}}$, i.e., $\tilde{\gamma}_u^{n,\text{init}} = q_n(y) + \frac{1}{N^*}\left(\tilde{\gamma}_u^{\text{init}}\right) - \frac{1}{{N^*}^2}\sum_{n\in\mathcal{N}_{\text{used}}} q_n(y_{\text{max}})$.

Thus, the problem of unequally re-distributing the SNR margin across tones is solved optimally. In a DSL system, when the noise spectrum changes and the system has stabilized after bit-swapping, the modem can once again re-distribute the margin to equalize the per-tone outage probabilities. If some tones are left with zero bits after bit-swapping, then the power from those tones can be equally distributed to the remaining used tones, which will increase the equivalent SNR margin, $\tilde{\gamma}_u^{\text{init}}$. This new SNR margin can then be re-distributed across tones using Algorithm 1.

5. NUMERICAL RESULTS

This section presents numerical results to illustrate the benefit of optimizing the per-tone margins in a DSL system, where the variations in channel SNR are primarily caused by noise variations. The noise

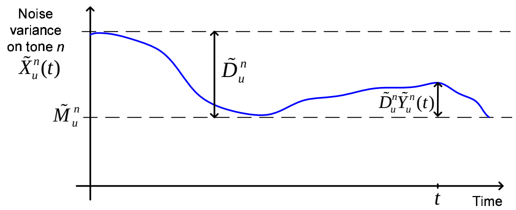

Fig. 1. Beta distribution model for the noise-variance.

variance on each tone is assumed to follow a Beta distribution since its flexibility can allow a good fit with the observed noise variances [2]. In practice, the channel SNR distribution can be empirically obtained by the modem and the Beta distribution is chosen in this paper simply for illustration of results. For user u on tone n, the random variable representing the noise variance in dBm, $\tilde{X}_u^n$, is modeled as $\tilde{X}_u^n = \tilde{M}_u^n + \tilde{D}_u^n\tilde{Y}_u^n$, where the constants $\tilde{M}_u^n$ and $(\tilde{M}_u^n + \tilde{D}_u^n)$, respectively, are the minimum and maximum values, in dBm, of the noise variance experienced by user u's receiver on tone n as illustrated in Fig. 1. $\tilde{Y}_u^n$ is a Beta distributed random variable, which is characterized using its *shape* parameters a_u^n and b_u^n, or equivalently using its mean, $\mu_{\tilde{Y}_u^n}$, and standard deviation, $\sigma_{\tilde{Y}_u^n}$. The observed noise variances on each tone are fitted to determine $\tilde{M}_u^n$, $\tilde{D}_u^n$, a_u^n, and b_u^n. The channel SNR random variable, G_u^n is then simply obtained as $G_u^n = |H_{u,u}^n|^2(\text{dB}) - \tilde{X}_u^n(\text{dBm})$.

An 4 km ADSL service is considered, and a 3 dB coding gain and 9.8 dB uncoded gap were used. The Beta distribution noise-spectrum model was used with -140 dBm/Hz AWGN as the minimum noise spectrum and an ANSI Noise A PSD [10], which is a mixture of 16 ISDN, 4 HDSL, and 10 ADSL disturbers, with a modified noise floor of -119 dBm/Hz, as the maximum noise spectrum. The shape parameters of the beta distributions were chosen to be $a_u^n = 4, b_u^n = 4$. Figure 2 illustrates the use of Algorithm 1 when the initialization-noise is equal to the average of the max and min noise-spectra. The initialization margin provided by the central management system is chosen to be 6 dB, which is typical in DSL. By re-distributing the available 6 dB margin across tones, the per-tone outage probability is equalized to a value much lower than the maximum outage probability across tones when the equal per-tone margin is applied. Therefore, when a sudden noise change occurs, the probability of the user going into an unstable state with a negative margin is reduced. In Fig. 2, the maximum outage probability across tones is reduced by an order of magnitude from 0.2 to 0.054. While the outage probabilities of some tones are increased, the increase is small compared to the overall reduction in the maximum outage probability. Figure 3 shows the unequal per-tone margins after re-distributing the available 6 dB margin. The tones which initially had a higher outage probability, because they experience larger noise-spectrum variations, are assigned margins that are much larger than 6 dB, while the margins on the tones that had the low outage probability are reduced by just 0.5 dB. Such simple re-distribution of the margin improves the overall outage probability of the user. Figures 2 and 3 show an improvement in the outage probabilities in the lower frequencies since the noise model assumed in the simulations allowed a larger variation in those frequencies w.r.t. the initialization noise-spectrum.

6. CONCLUSIONS

An algorithm to optimally distribute the available SNR margin across tones in a multicarrier communication system was presented. The algorithm can be used by modems to ensure better robustness against

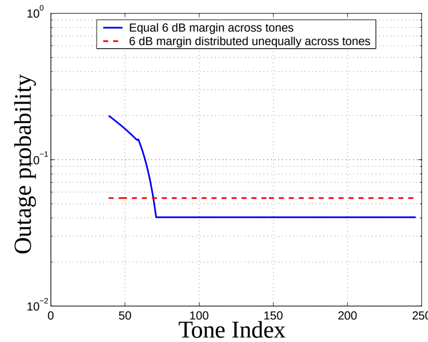

Fig. 2. Equalization of the per-tone outage probabilities by re-distributing the available 6 dB margin across tones.

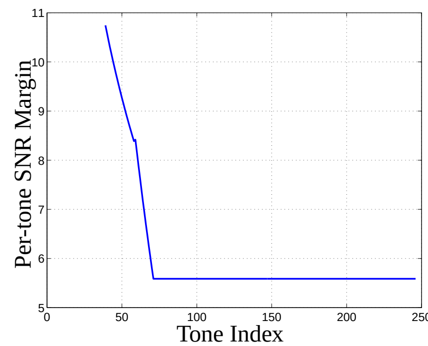

Fig. 3. New per-tone margins after re-distributing the 6 dB margin.

abrupt channel-SNR changes. Such re-distribution of margin across tones is especially important for multicarrier systems that do not have bit and energy adaptation procedures during online operation of the modem or that have too slow adaptation procedures. Thus, optimization of the per-tone margins provides improved QoS compared to allocating equal per-tone margins.

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